# Energy-based damage index for steel structures

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**Abstract.** Ample research effort has been oriented into developing damage indices with the aim of estimating in a reasonable manner the consequences, in terms of structural damage and deterioration, of severe plastic cycling. Although several studies have been devoted to calibrate damage indices for steel and reinforced concrete members; currently, there is a challenge to study and calibrate the use of such indices for the practical evaluation of complex structures. The aim of this paper is to introduce an energy-based damage index for multi-degree-of-freedom steel buildings that accounts explicitly for the effects of cumulative plastic deformation demands. The model has been developed by complementing the results obtained from experimental testing of steel members with those derived from analytical studies regarding the distribution of plastic demands on several steel frames designed according to the Mexico City Building Code. It is concluded that the approach discussed herein is a promising tool for practical structural evaluation of framed structures subjected to large energy demands.

**Keywords:** energy-based damage index; plastic hysteretic energy; cumulative plastic deformation demands; steel frames.

## 1. Introduction

Within current seismic design formats, the maximum inter-story drift and ductility demands are targeted as performance parameters to achieve adequate damage control in earthquake-resistant structures. However, there is ample evidence that in some cases, the structural performance of structures subjected to long duration ground motions cannot be adequately characterized through their maximum deformation demands (Fajfar 1992, Cosenza and Manfredi 1996, Terán-Gilmore 1996, Fajfar and Krawinkler 1997, Rodríguez and Ariztizabal 1999, Bojórquez and Ruiz 2004, Arroyo and Ordaz 2007, Hancook and Boomer 2006, Terán-Gilmore and Jirsa 2007), in such manner that the effect of cumulative plastic deformation demands should be accounted explicitly during seismic design.

The effect of cumulative plastic deformation demands can be considered through the use of energy concepts; particularly through the plastic dissipated hysteretic energy demand. The use of energy for this purpose was initially discussed by Housner (1956), and has been used by several researchers to propose energy-based methodologies that aim at providing earthquake-resistant structures with an energy dissipating capacity larger or equal than its corresponding demand (Akiyama 1985, Akbas et al. 2001, Choi and Kim 2006, Bojórquez *et al.* 2008a).

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Due to the limitations of the maximum deformation as the principal indicator of structural damage, several damage indices that account for the plastic dissipated hysteretic energy have been formulated to better represent the consequences, in terms of structural damage and deterioration, of severe plastic cycling. Most research has been devoted to calibrate damage indices for steel and reinforced concrete members (Krawinkler and Zohrei 1983, Park and Ang 1985, Bozorgnia and Bertero 2001, Teran and Jirsa 2005, Rodriguez and Padilla 2008). However, there is a challenge to study and calibrate the use of such indices for the practical structural evaluation of complex structures. Within this context, an energy-based damage index which explicitly accounts for the effects of cumulative plastic deformation demands on multi-degree-of-freedom (MDOF) steel frames is introduced herein.

## 2. Energy-based damage index

Energy-based methodologies are focused at providing structures with energy dissipating capacities that are larger or equal than their expected energy demands (Akiyama 1985, Uang and Bertero 1990). The design requirements of an earthquake-resistant structure in these terms can be formulated as

$$Energy\ Capacity \geq Energy\ Demand \tag{1}$$

Among all the energies absorbed and dissipated by a structure, the plastic hysteretic energy $E_H$ is clearly related to structural damage. $E_H$ can be physically interpreted by considering that it is equal to the total area under all the hysteresis loops that a structure undergoes during a ground motion. Therefore, it is convenient to express Eq. (1) in terms of plastic hysteretic energy

$$E_{HC} \geq E_{HD} \tag{2}$$

where $E_{HC}$ is the plastic hysteretic energy capacity and $E_{HD}$ is the plastic hysteretic energy demand. Eq. (2) can be reformulated as an energy-based damage index

$$I_{DE} = \frac{E_{HD}}{E_{HC}} \leq 1 \tag{3}$$

In Eq. (3), the performance level or condition where $E_{HD}$ equals $E_{HC}$ will be considered as the failure of the system. Hence, while $I_{DE} = 1$ corresponds to failure of the structural system; a value of zero implies no structural damage (elastic behavior implies no structural damage). From a physical point of view, this equation represents a balance between the structural capacity and demand in terms of energy. In this sense, this formulation follows the direction initially established by Housner in (1956) for an energy-based design.

According to Eq. (3), structural damage depends on the balance between the plastic hysteretic energy capacity and demand on the structure. While the plastic hysteretic energy demand can be obtained through dynamic analysis, a challenge exists to define the plastic hysteretic energy capacity of a structure. Nevertheless, flexural plastic behavior is usually concentrated at the ends of the structural members that make up a frame; and in the particular case of W steel shapes, in the flanges. The plastic hysteretic energy capacity of a steel member that forms part of a structural frame can be estimated as follows (Akbas *et al.* 2001)

$$E_{HCm} = 2Z_f f_y \theta_{pa} \tag{4}$$

where $Z_f$ is the section modulus of the flanges of the steel member; $f_y$, the yield stress; and $\theta_{pa}$, its cumulative plastic rotation capacity. While the above equation considers that plastic energy is dissipated exclusively through plastic behavior at both ends of a steel member, the definition of cumulative plastic rotation is schematically illustrated in Fig. 1.

Eq. (4) can be used together with Eq. (3) to evaluate the level of structural damage in steel members. However, for damage evaluation purposes it is convenient to normalize the hysteretic energy $E_H$ as follows (Krawinkler and Nassar 1992, Terán-Gilmore and Simon 2006)

$$E_N = \frac{E_H}{F_y \delta_y} \tag{5}$$

where $F_y$ and $\delta_y$ are the strength and displacement at first yield, respectively. Eq. (3) can be expressed in terms of $E_N$ as follows

$$I_{DEN} = \frac{E_{ND}}{E_{NC}} \leq 1 \tag{6}$$

where the parameters involved in Eq. (6) have the same meaning as those used in Eq. (3). The advantage of formulating the problem in terms of $E_N$ is that this is a more stable parameter, and in quantitative terms it can easily be used for practical purposes. In other words, the energy-based damage index proposed herein corresponds to the ratio between the normalized hysteretic energy demand and normalized hysteretic energy capacity, and the condition of failure is assumed to be $I_{DEN}$ equal to one.

In the case of MDOF steel structures, the principal challenge for the practical use of Eq. (6) is the definition of the energy capacity of the structure in terms of that of its structural members. Through the consideration that in regular steel frames the energy is dissipated exclusively by the beams (which is an appropriate hypothesis for strong column-weak beam structural systems), the energy capacity of these

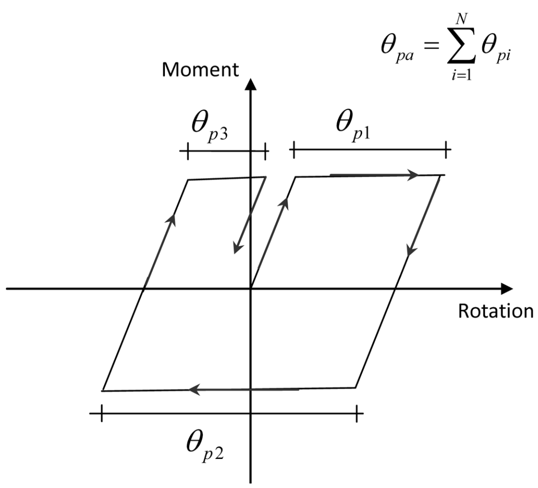



Fig. 1 Definition of cumulative plastic rotation.

systems can be estimated as (Bojórquez *et al*. 2008a)

$$E_{NC} = \frac{\sum_{i=1}^{N_s}(2N_B Z_f F_y \theta_{pa} F_{EHi})}{C_y D_y W} \tag{7}$$

where $N_S$ and $N_B$ are the number of stories and bays in the building, respectively; $F_{EHi}$ an energy participation factor that accounts for the different contribution of each story to the energy dissipation capacity of a frame; $W$ is the total weight of the structure; and finally, $C_y$ and $D_y$, the seismic coefficient and displacement at first yield, which can be obtained, as shown in Fig. 2, from the capacity curve of the frames.

Eq. (7) shows the role of the cumulative plastic rotation capacity of the structural members in the total energy dissipation capacity of a frame. Fig. 3 shows a wide range of $\theta_{pa}$ values collected by Akbas

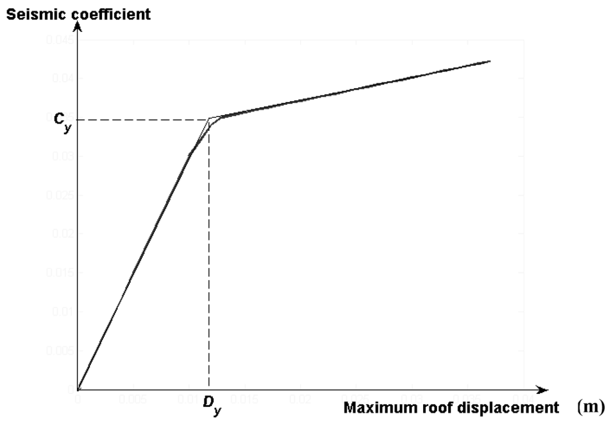



Fig. 2 Evaluation of $C_y$ and $D_y$ from capacity curve

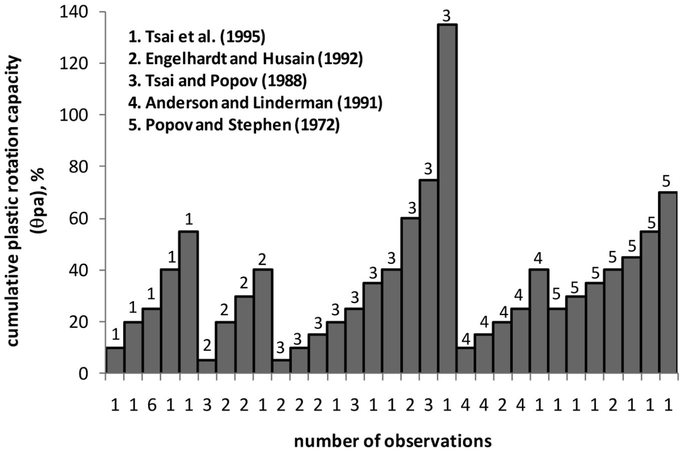



Fig. 3 Cumulative plastic rotation capacity of steel members (Akbas 1997)

(1997) from experimental testing of steel members subjected to cyclic loading. Based on the results collected by Akbas(1997), Bojórquez *et al*. (2008b) found that the cumulative plastic rotation capacity of steel members is well represented by a lognormal probability density function with a median value equal to 0.23.

Although the selection of a value of $\theta_{pa}$ to estimate through Eq. (7) the plastic hysteretic energy capacity of a steel frame is a difficult task, it should be emphasized that current experimental evidence provides a reasonable basis for such selection. Particularly, the median value reported by Bojórquez *et al*. (2008b) and based in the experimental results collected by Akbas (1997) will be used herein (0.23). To provide some context to this value, it should be mentioned that Calderoni and Rinaldi (2000, 2002) reported ultimate plastic rotation capacities close to 0.04 for ductile steel beams. The ratio between the ultimate rotation capacity of 0.04 and the cumulative rotation capacity of 0.23 is close to 17%; value that is very close to the average value of 18% reported by Brescia *et al*. (2009) for the ratio between the energy absorbed during monotonic testing and that dissipated under cyclic testing of twelve ductile steel members.

## 3. Energy and damage distribution in regular steel frames

To estimate the contribution of the different structural members to the total plastic hysteretic energy capacity of MDOF frames, it is usually necessary to assume a distribution of plastic energy dissipation along height. For instance, while Akbas *et al*. (2001) proposed a linear distribution, recent studies suggest that if energy dissipation is concentrated in the beams of a frame, a lognormal distribution represents a better approximation (Bojórquez *et al*. 2008a). A plastic hysteretic energy participation factor ($F_{EH}$) needs to be established to account properly within Eq. (7) for the different contribution of each story to the total energy dissipating capacity of a building. In particular, $F_{EH}$ can be formulated so that it evaluates the percentage of the ultimate energy capacity that a story dissipates during the ground motion (the critical stories contributing their full energy dissipating capacity, fact that is expressed through a unitary value for $F_{EH}$). Normally, an expression to describe the variation of $F_{EH}$ along height is derived from plastic energy demand distributions estimated analytically in prototype frames and buildings. From extensive statistical studies of eight steel moment-resisting frames subjects to several long duration ground motions, $F_{EHi}$ was characterized by Bojórquez *et al*. (2008a) with the following expression

$$F_{EH} = m?(F_{EH}{}^*, 1) \tag{8}$$

where

$$F_{EH}{}^* = \frac{1}{(-0.0645\mu + 2.82)h/H}\exp\left\{-\frac{1}{2}\left[\frac{\ln(h/H) - \ln(0.031\mu + 0.3461)}{0.06\mu + 0.39}\right]^2\right\}$$

Fig. 4 shows the evolution of the value of $F_{EH}$ along height for buildings with increasing ductility. Note the increase in the values of $F_{EH}$, particularly in the upper stories of the frames, for increasing ductility. This evolution indicates that the beams located at the upper stories tend to contribute more to the energy dissipating capacity of the frames as the global ductility in the frames increases.

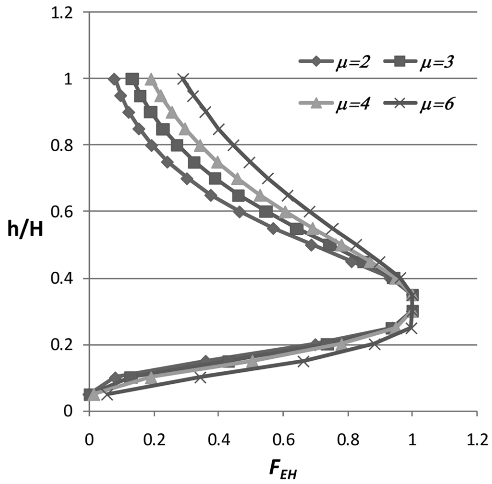



Fig. 4 Evolution of $F_{EH}$ with increasing global ductility

Although this manner of establishing $F_{EH}$ provides a reasonable approximation on how the structural members contribute to the total plastic hysteretic energy capacity of framed structures; it has the limitation of not considering the actual energy dissipating capacity of individual members, which may vary from story to story, and from bay to bay. This implies, as will be discussed in the next section, the need to address the damage distribution along height in lieu of the dissipated hysteretic energy configuration.

### 3.1 Numerical characterization of damage distribution along height

The evaluation of a factor to characterize a damage distribution along height ($F_D$) is based herein on the energy damage models discussed in the previous sections. Six regular steel frames designed according to the Mexico City Building Code were subjected to 23 soft-soil long duration ground motions recorded in the Lake Zone of Mexico City and exhibiting a dominant period ($T_s$) of two seconds. Particularly, all motions were recorded during seismic events with magnitudes of seven or larger, and having epicenters located at distances of 300 km or more from Mexico City. The frames, which were assumed to be used for office occupancy, have three bays and a number of levels that range from four to eighteen. The bay and inter-story dimensions are those indicated in Fig. 5. The frames were designed for ductile detailing. A36 steel and W sections were used for the beams and columns of the frames. A two dimensional, lumped plasticity nonlinear model of each frame was prepared and analyzed with the program RUAUMOKO (Carr 2002). For this purpose, an elasto-plastic model with 3% strain-hardening was used to represent the cyclic behavior of the transverse sections located at both ends of the steel beams and columns. As discussed by Bojórquez and Rivera (2008), this model provides a good approximation to the actual hysteretic behavior of steel members. While the slabs were modeled as rigid in-plane diaphragms, the columns in the first story were modeled as clamped at their bases and the beam-column connections were assumed to be rigid. Second order effects were explicitly considered. Time-history analyses were carried out for each frame. In the analyses, the first two modes of vibration were assigned 3% of critical damping. Relevant characteristics for each frame, such as the fundamental period of vibration ($T_1$), and the seismic coefficient and displacement at first yield ($C_y$ and

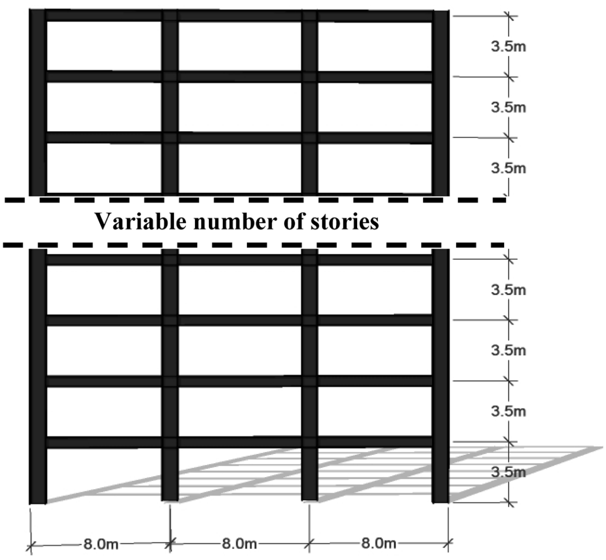



Fig. 5 Geometrical characteristics of steel frames

$D_y$) are shown in Table 1 (the latter two values were established from static nonlinear analyses). Note that the frames exhibit a wide range of periods. Furthermore, while Table 2 summarizes the member sizes for all frames under consideration, Table 3 summarizes the principal characteristics of the seismic records under consideration, and Fig. 6 shows their ground motion duration established according to Trifunac and Brady (1975). This duration is defined as the time interval delimited by the instants of time at which the 5% and 95% of the Arias Intensity (Arias 1970) occurs. Note that the average duration of the records equals 73.1 sec. The very long duration of the motions helps to illustrate the importance of cumulative demand parameters during seismic design.

For the purpose of obtaining the damage distribution along height for a given frame, the frame was subjected to all the records scaled up according to a target spectral acceleration evaluated at its fundamental period of vibration $Sa(T_1)$. Several target spectral accelerations were used in this manner until the median value of the damage index in the critical story of the frame reached the value of one. The value of one represents the threshold associated to structural failure of the MDOF steel frame. It should be mentioned that all beams within a story exhibited similar level of damage, in such manner that it is possible to assign a unique value of damage to that story. Bojórquez *et al*. (2006) have discussed the pertinence of this approach to represent structural failure. As discussed before, a median

Table 1 Relevant characteristics of the steel frames

| Frame | Number of Stories | $T_1$ $(s)$ | $C_y$ | $D_y$ (m) |
|---|---|---|---|---|
| F4 | 4 | 0.90 | 0.45 | 0.136 |
| F6 | 6 | 1.07 | 0.42 | 0.174 |
| F8 | 8 | 1.20 | 0.38 | 0.192 |
| F10 | 10 | 1.37 | 0.36 | 0.226 |
| F14 | 14 | 1.91 | 0.25 | 0.30 |
| F18 | 18 | 2.53 | 0.185 | 0.41 |

Table 2 Summary of the member sizes of the steel frames (Continued)

| FRAME | F4 | F6 | F8 | F10 | F14 | F18 |
|---|---|---|---|---|---|---|
| Number of Stories | 4 | 6 | 8 | 10 | 14 | 18 |
| Internal columns | | | | | | |
| Story 1 | $W21 \times 122$ | $W30 \times 173$ | $W36 \times 210$ | $W36 \times 280$ | $W36 \times 328$ | $W36 \times 359$ |
| Story 2 | $W21 \times 122$ | $W30 \times 173$ | $W36 \times 210$ | $W36 \times 280$ | $W36 \times 328$ | $W36 \times 359$ |
| Story 3 | $W21 \times 111$ | $W30 \times 148$ | $W36 \times 194$ | $W36 \times 245$ | $W36 \times 280$ | $W36 \times 359$ |
| Story 4 | $W21 \times 111$ | $W30 \times 148$ | $W36 \times 194$ | $W36 \times 245$ | $W36 \times 280$ | $W36 \times 359$ |
| Story 5 | | $W30 \times 124$ | $W36 \times 170$ | $W36 \times 210$ | $W36 \times 280$ | $W36 \times 328$ |
| Story 6 | | $W30 \times 124$ | $W36 \times 170$ | $W36 \times 210$ | $W36 \times 280$ | $W36 \times 328$ |
| Story 7 | | | $W36 \times 160$ | $W36 \times 182$ | $W36 \times 245$ | $W36 \times 280$ |
| Story 8 | | | $W36 \times 160$ | $W36 \times 182$ | $W36 \times 245$ | $W36 \times 280$ |
| Story 9 | | | | $W36 \times 150$ | $W36 \times 210$ | $W36 \times 245$ |
| Story 10 | | | | $W36 \times 150$ | $W36 \times 210$ | $W36 \times 245$ |
| Story 11 | | | | | $W36 \times 182$ | $W36 \times 210$ |
| Story 12 | | | | | $W36 \times 182$ | $W36 \times 210$ |
| Story 13 | | | | | $W36 \times 150$ | $W36 \times 182$ |
| Story 14 | | | | | $W36 \times 150$ | $W36 \times 182$ |
| Story 15 | | | | | | $W36 \times 150$ |
| Story 16 | | | | | | $W36 \times 150$ |
| Story 17 | | | | | | $W36 \times 150$ |
| Story 18 | | | | | | $W36 \times 150$ |
| External columns | | | | | | |
| Story 1 | $W18 \times 97$ | $W27 \times 146$ | $W36 \times 194$ | $W36 \times 280$ | $W36 \times 328$ | $W36 \times 359$ |
| Story 2 | $W18 \times 97$ | $W27 \times 146$ | $W36 \times 194$ | $W36 \times 280$ | $W36 \times 328$ | $W36 \times 359$ |
| Story 3 | $W18 \times 86$ | $W27 \times 129$ | $W36 \times 182$ | $W36 \times \times 245$ | $W36 \times 280$ | $W36 \times 359$ |
| Story 4 | $W18 \times 86$ | $W27 \times 129$ | $W36 \times 182$ | $W36 \times 245$ | $W36 \times 280$ | $W36 \times 359$ |
| Story 5 | | $W27 \times 114$ | $W36 \times 160$ | $W36 \times 210$ | $W36 \times 280$ | $W36 \times 328$ |
| Story 6 | | $W27 \times 114$ | $W36 \times 160$ | $W36 \times 210$ | $W36 \times 280$ | $W36 \times 328$ |
| Story 7 | | | $W36 \times 135$ | $W36 \times 182$ | $W36 \times 245$ | $W36 \times 280$ |
| Story 8 | | | $W36 \times 135$ | $W36 \times 182$ | $W36 \times 245$ | $W36 \times 280$ |
| Story 9 | | | | $W36 \times 150$ | $W36 \times 210$ | $W36 \times 245$ |
| Story 10 | | | | $W36 \times 150$ | $W36 \times 210$ | $W36 \times 245$ |
| Story 11 | | | | | $W36 \times 182$ | $W36 \times 210$ |
| Story 12 | | | | | $W36 \times 182$ | $W36 \times 210$ |
| Story 13 | | | | | $W36 \times 150$ | $W36 \times 182$ |
| Story 14 | | | | | $W36 \times 150$ | $W36 \times 182$ |
| Story 15 | | | | | | $W36 \times 150$ |
| Story 16 | | | | | | $W36 \times 150$ |
| Story 17 | | | | | | $W36 \times 150$ |
| Story 18 | | | | | | $W36 \times 150$ |

Table 2. Continued...

| FRAME | F4 | F6 | F8 | F10 | F14 | F18 |
|---|---|---|---|---|---|---|
| Number of Stories | 4 | 6 | 8 | 10 | 14 | 18 |
| Beams | | | | | | |
| Story 1 | $W16\times67$ | $W18\times71$ | $W21\times83$ | $W21\times68$ | $W21\times93$ | $W21\times101$ |
| Story 2 | $W16\times57$ | $W18\times76$ | $W21\times93$ | $W21\times93$ | $W21\times93$ | $W21\times101$ |
| Story 3 | $W16\times45$ | $W18\times76$ | $W21\times93$ | $W21\times101$ | $W21\times111$ | $W21\times111$ |
| Story 4 | $W16\times40$ | $W16\times67$ | $W21\times83$ | $W21\times101$ | $W21\times111$ | $W21\times111$ |
| Story 5 | | $W16\times50$ | $W18\times71$ | $W21\times101$ | $W21\times111$ | $W21\times111$ |
| Story 6 | | $W16\times45$ | $W18\times65$ | $W21\times93$ | $W21\times101$ | $W21\times101$ |
| Story 7 | | | $W18\times55$ | $W21\times73$ | $W21\times93$ | $W21\times101$ |
| Story 8 | | | $W18\times46$ | $W21\times68$ | $W21\times83$ | $W21\times93$ |
| Story 9 | | | | $W21\times57$ | $W21\times83$ | $W21\times93$ |
| Story 10 | | | | $W21\times50$ | $W21\times73$ | $W21\times83$ |
| Story 11 | | | | | $W21\times73$ | $W21\times83$ |
| Story 12 | | | | | $W21\times62$ | $W21\times73$ |
| Story 13 | | | | | $W21\times62$ | $W21\times73$ |
| Story 14 | | | | | $W21\times57$ | $W21\times62$ |
| Story 15 | | | | | | $W21\times62$ |
| Story 16 | | | | | | $W21\times62$ |
| Story 17 | | | | | | $W21\times57$ |
| Story 18 | | | | | | $W21\times57$ |

value of 0.23 was assigned to the cumulative plastic rotation capacity of the beams.

The results obtained from the nonlinear dynamic analyses of the frames are illustrated in Fig. 7. In this figure, h/H represents the height of a story relative to the ground normalized with respect to the total height of the structure (H). Only the median damage values are plotted. As suggested by Bojórquez *et al*. (2008a), the median damage value along height is well represented by a lognormal distribution (note that this is valid for the different frames in spite of their varying number of stories). The continuous black line in the figure corresponds to the lognormal distribution fitted to the data through a regression analysis. It is observed that damage tends to concentrate around h/H of 0.4. Based on the results derived from the nonlinear dynamic and regression analyses, the following expression was established to describe the variation of damage along height

$$F_D = \frac{1}{2.33h/H}\exp\left\{-\frac{1}{2}\left[\frac{(\ln(h/H)-\ln(0.52))}{0.49}\right]^2\right\} \tag{9}$$

Eq. (9) can be used for regular steel framed structures designed according to a capacity design approach (whose response is characterized by the concentration of plastic demands in the beams) and subjected to long duration ground motions. The value of $F_D$ derived from Eq. (9) can be used in Eq. (7) in lieu of the energy participation factor ($F_{EH}$) that accounts for the different contribution of each story

Table 3 Ground motion records

| Record | Date | Magnitude | Station | *PGA* (cm/s²) | *PGV* (cm/s) |
|---|---|---|---|---|---|
| 1 | 25/04/1989 | 6.9 | Alameda | 45.0 | 15.6 |
| 2 | 25/04/1989 | 6.9 | Garibaldi | 68.0 | 21.5 |
| 3 | 25/04/1989 | 6.9 | SCT | 44.9 | 12.8 |
| 4 | 25/04/1989 | 6.9 | Sector Popular | 45.1 | 15.3 |
| 5 | 25/04/1989 | 6.9 | Tlatelolco TL08 | 52.9 | 17.3 |
| 6 | 25/04/1989 | 6.9 | Tlatelolco TL55 | 49.5 | 17.3 |
| 7 | 14/09/1995 | 7.3 | Alameda | 39.3 | 12.2 |
| 8 | 14/09/1995 | 7.3 | Garibaldi | 39.1 | 10.6 |
| 9 | 14/09/1995 | 7.3 | Liconsa | 30.1 | 9.62 |
| 10 | 14/09/1995 | 7.3 | Plutarco Elías Calles | 33.5 | 9.37 |
| 11 | 14/09/1995 | 7.3 | Sector Popular | 34.3 | 12.5 |
| 12 | 14/09/1995 | 7.3 | Tlatelolco TL08 | 27.5 | 7.8 |
| 13 | 09/10/1995 | 7.5 | Cibeles | 14.4 | 4.6 |
| 14 | 09/10/1995 | 7.5 | Córdoba | 24.9 | 8.6 |
| 15 | 09/10/1995 | 7.5 | Liverpool | 17.6 | 6.3 |
| 16 | 09/10/1995 | 7.5 | Plutarco Elías Calles | 19.2 | 7.9 |
| 17 | 11/01/1997 | 6.9 | CU Juárez | 16.2 | 5.9 |
| 18 | 11/01/1997 | 6.9 | Centro urbano Presidente Juárez | 16.3 | 5.5 |
| 19 | 11/01/1997 | 6.9 | García Campillo | 18.7 | 6.9 |
| 20 | 11/01/1997 | 6.9 | Plutarco Elías Calles | 22.2 | 8.6 |
| 21 | 11/01/1997 | 6.9 | Est. # 10 Roma A | 21.0 | 7.76 |
| 22 | 11/01/1997 | 6.9 | Est. # 11 Roma B | 20.4 | 7.1 |
| 23 | 11/01/1997 | 6.9 | Tlatelolco TL55 | 13.4 | 6.5 |

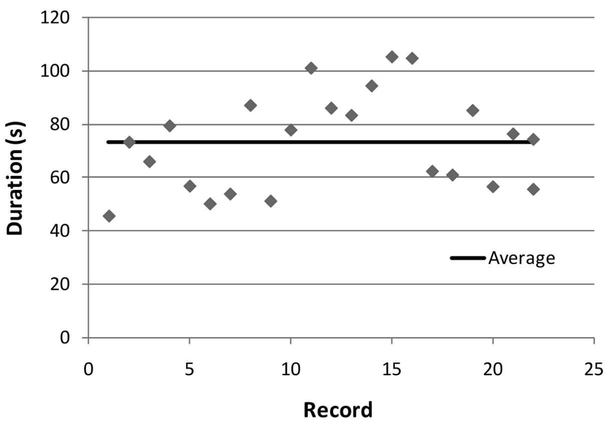



Fig. 6 Ground motion duration of selected records

to the energy dissipating capacity of the frame. Once the value of $E_{NC}$ is established, it can be used in Eq. (6) to evaluate the structural performance of a frame.

The use of $F_D$ in lieu of $F_{EH}$ has some advantages. First, damage is a more robust demand parameter

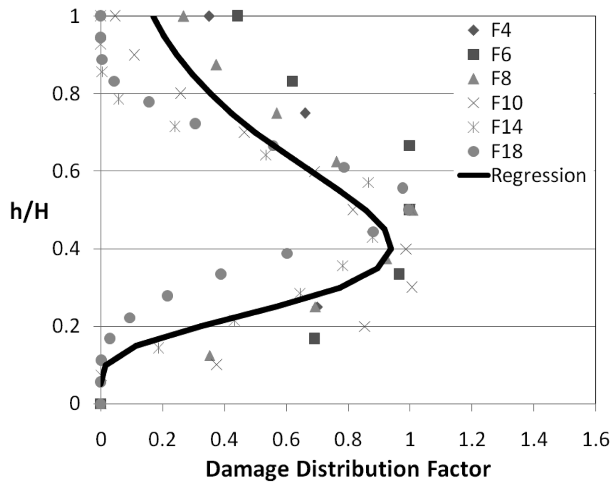



Fig. 7 Median damage distribution along height for all structures and records

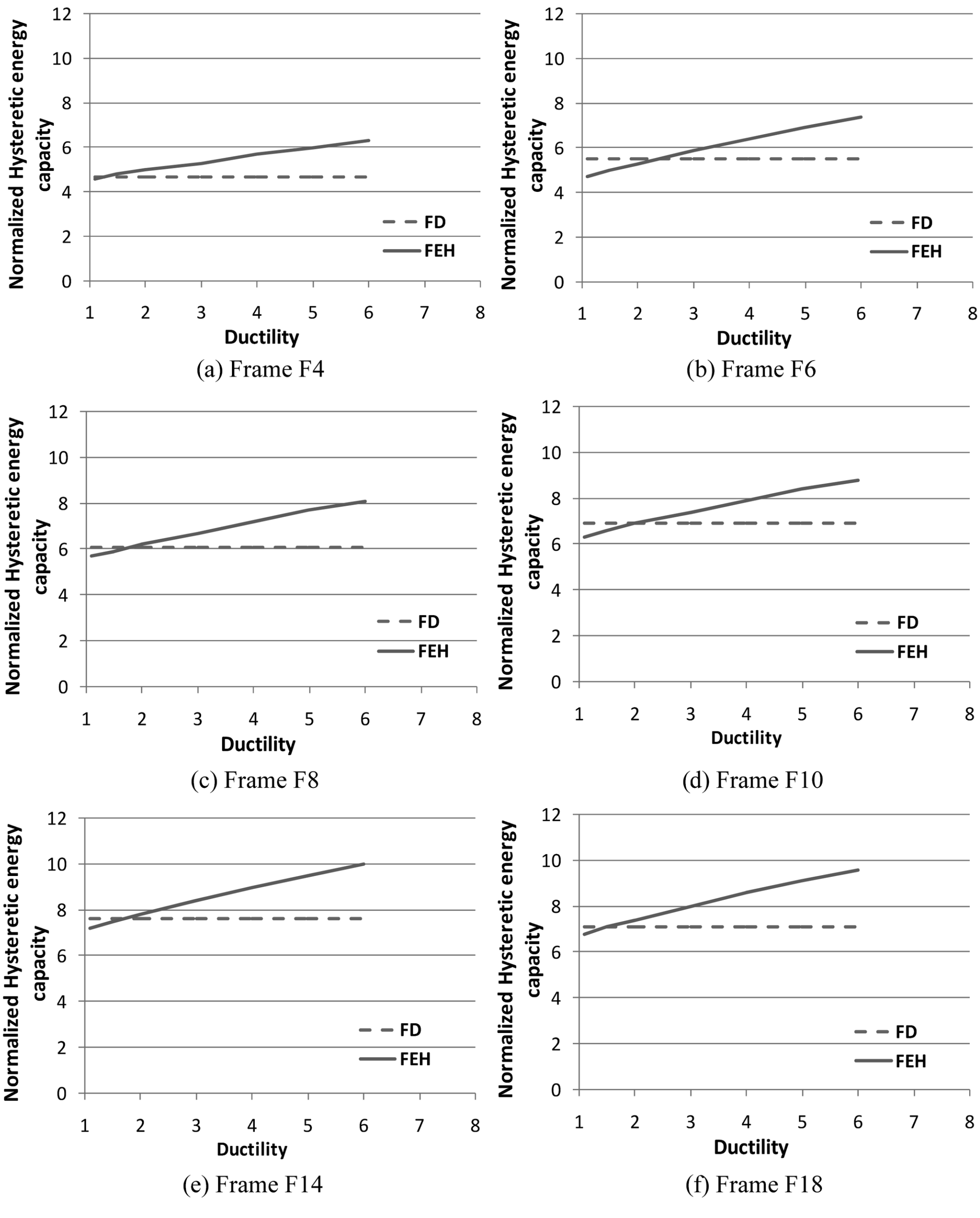
than plastic energy. Particularly, $F_D$ exhibits a weaker dependence with respect to the ductility demand in the frames, in such a manner that unlike Eq. (8), Eq. (9) does not exhibit dependence with respect to ductility. Second, it is possible that two stories having different energy dissipating capacities exhibit similar energy demands. While under these circumstances $F_{EH}$ could not indicate the different level of damage in both stories, $F_D$ is able to do so. Finally, as mentioned previously, $F_D$ is a better indicator of inter-story structural damage in case the actual energy dissipating capacity of individual members varies from story to story, and from bay to bay.

### *3.2 Evaluation of normalized hysteretic energy capacity: $F_{EH}$ versus $F_D$*

The advantages of using the damage distribution through height factor (Eq. (9)) in lieu of the hysteretic energy distribution through height factor (Eq.(8)) is discussed next. It should be mentioned that the hysteretic energy distribution along height factor has been used successfully in previous papers to assess damage in steel frames (*e.g.*, Bojórquez *et al.* 2008a, b).

While the value of $F_D$ only depends on the height with respect to the ground of the story where the factor is estimated, the evaluation of $F_{EH}$ requires also the knowledge of the maximum ductility demand associated to the failure of the frame. Although this maximum ductility demand or target demand could be estimated for systems subjected to severe cumulative plastic deformation demands (Bojórquez *et al.* 2009), this results in added complication to the evaluation process in such manner that the use of $F_D$ should be studied in terms of its technical pertinence during damage evaluation.

The global normalized hysteretic energy capacity of the steel frames was estimated using in Eq. (7) both factors under consideration: $F_{EH}$ and $F_D$. For this purpose, it was assumed that the cumulative plastic rotation capacity at the ends of the beams is equal to 0.23. Fig. 8 compares both estimates of hysteretic energy capacity for all frames. For any specific frame, the energy capacity derived from $F_D$ is constant throughout the range of maximum ductilities under consideration. In contrast, the capacity derived from $F_{EH}$ increases with increasing ductility. Note that for small and moderate ductility demands (ranging from one to three), the energy capacities derived from both factors are practically the same, in such way that an evaluation procedure that uses either one of them will yield similar estimates




(a) Frame F4 (b) Frame F6

(c) Frame F8 (d) Frame F10

(e) Frame F14 (f) Frame F18

Fig. 8 Comparison of the normalized hysteretic energy capacity of the frames using $F_{EH}$ and $F_D$

of damage. Within this context, it is important to mention that studies carried out by Bojórquez *et al.* (2009) suggest that ductile structures can't undergo ductility demands larger than three during severe ground motions exhibiting high energy contents. Moreover, Fig. 9 compares explicitly the values of $F_{EH}$ and $F_D$ for Frame F8 and ductilities of two and three. It is observed that the variation along height

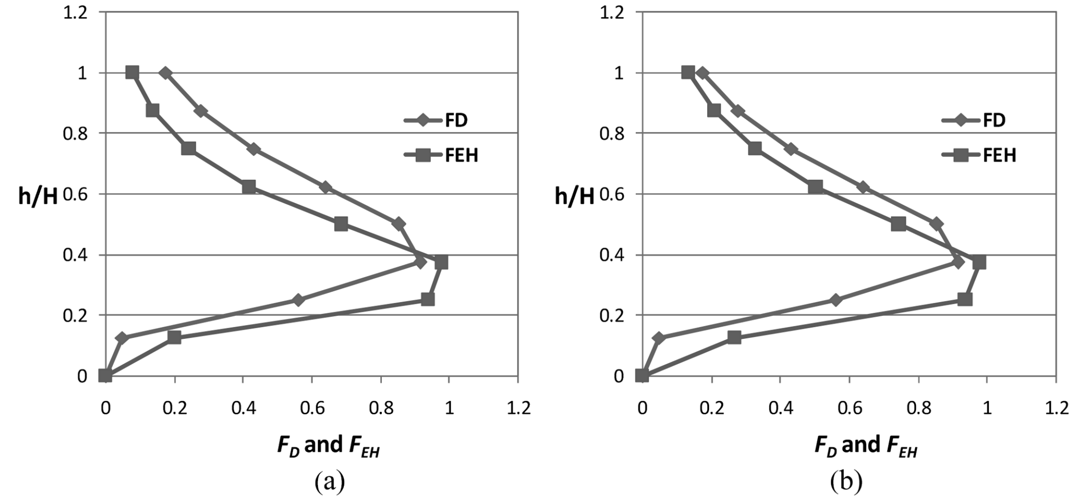



Fig. 9 Variation of $F_{EH}$ and $F_D$ along height for ductilities of 2 and 3, Frame F8

of $F_{EH}$ and $F_D$ is similar for both values of ductility, in such a manner that for the sample frames, both parameters can be used for reasonable evaluation of their normalized hysteretic energy capacity.

Fig. 10 shows the dependence of the energy capacity of the frames estimated from $F_D$, with respect to their number of stories and fundamental period of vibration period. As shown, the normalized hysteretic energy capacity increases with the number of stories and the structural period of the frames. It should be emphasized that the energy capacities shown in Fig. 10 are those that the frames exhibit at failure.

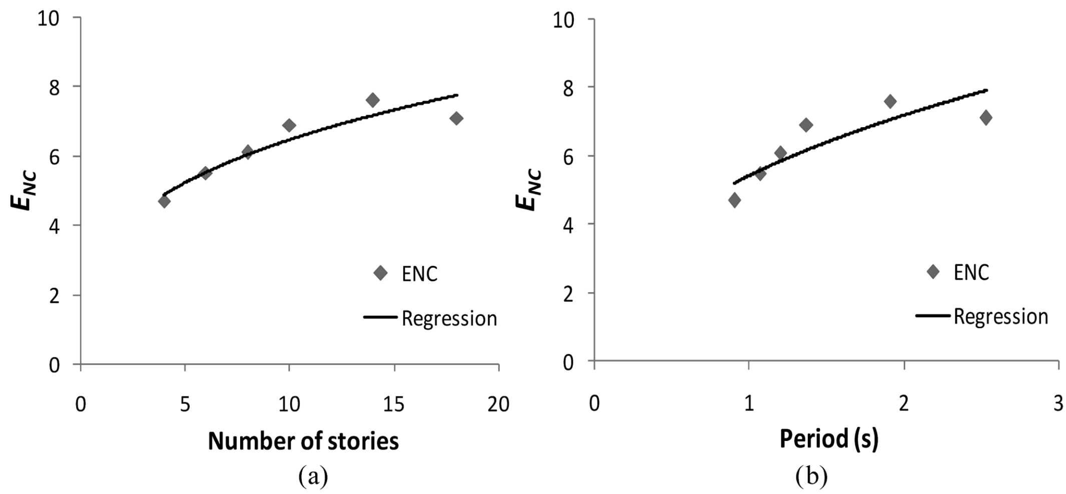



Fig. 10 Variation of the normalized hysteretic energy capacity with: (a) Number of stories; (b) Structural period

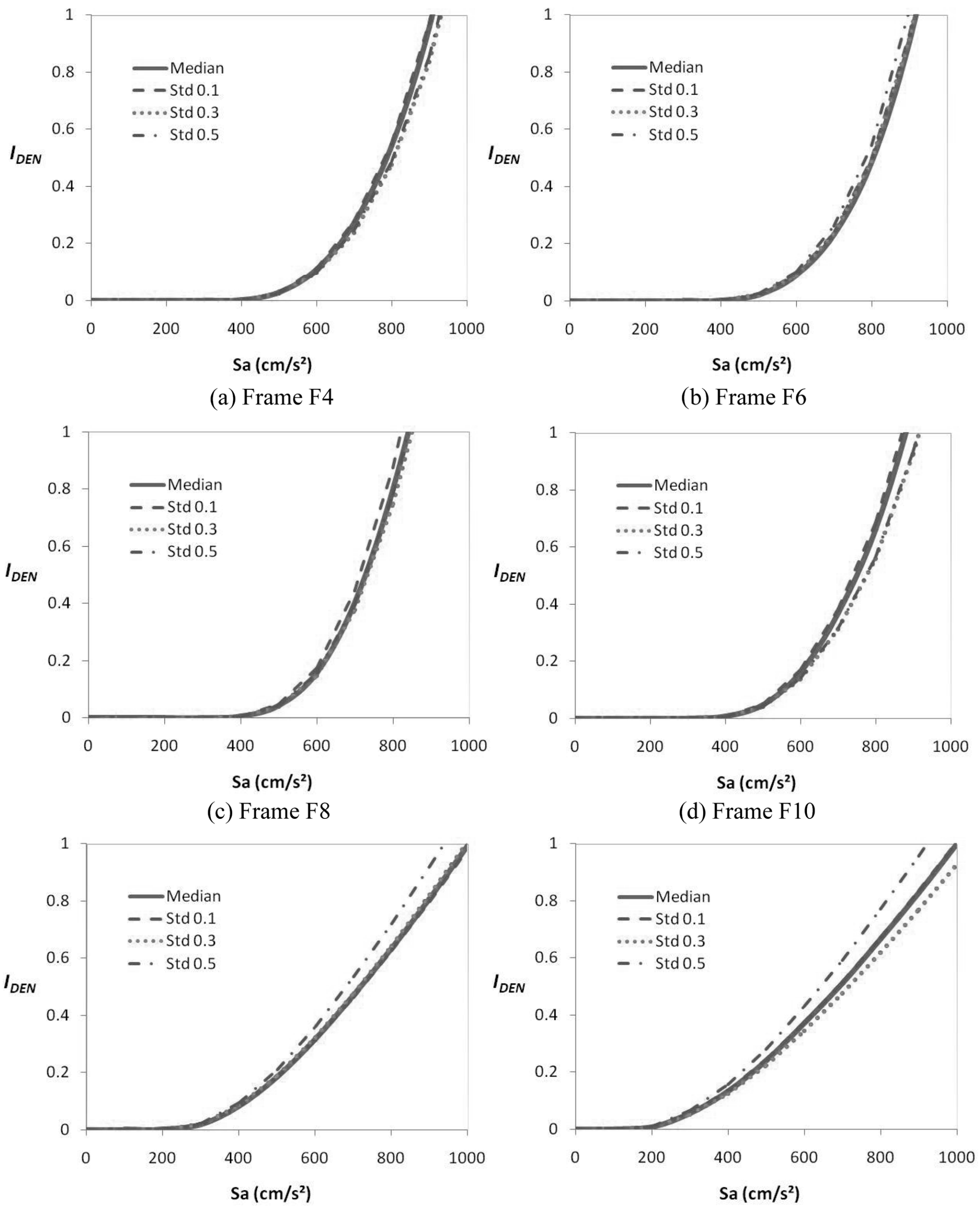
## 4. Effect of uncertainty in the cumulative plastic rotation capacity on the evaluation of structural damage

The structural damage in the frames under consideration was evaluated through Eq. (7) and $F_D$ to assess the effect of explicitly considering the uncertainty of the cumulative plastic rotation capacity of the structural members. A lognormal probability density function with a median value of 0.23 was used to describe the variation of the cumulative plastic rotation capacity at the ends of the beams (Bojórquez *et al*. 2008b). For illustration purposes, four standard deviations of the natural logarithm were considered: 0, 0.1, 0.3 and 0.5. A standard deviation of zero corresponds to the mean values. While it was assumed that the value of $\theta_{pa}$ varied in height according to the lognormal density function, the value of $\theta_{pa}$ for all beams within a story was considered equal. The $\theta_{pa}$ values obtained for all frames and a standard deviation of 0.1 are summarized in Table 4.

The influence of the uncertainty of the cumulative plastic rotation capacity is illustrated through the results obtained from incremental dynamic analyses of all frames under consideration. For this purpose, the frames were subjected to the ground motions included in Table 3 scaled up in such manner as to achieve the same spectral ordinate at the period corresponding to the first mode of vibration of each particular frame. A wide range of motion intensities were considered for this purpose. Fig. 11 shows and compares the median values of $I_{DEN}$ obtained from Eq. (6) for the steel frames. The horizontal axis considers the different intensity levels quantified through the spectral acceleration associated to the first mode of vibration. The comparison suggests that there is no significant influence of the level of uncertainty of $\theta_{pa}$ in the damage estimates for the frames. In general, the damage estimates corresponding to the different levels of uncertainty is quite similar for each particular frame. It can be

Table 4 Simulated $\theta_{pa}$ values, median of 0.23 and standard deviation of 0.1

| FRAME<br>Story | F4 | F6 | F8 | F10 | F14 | F18 |
|---|---|---|---|---|---|---|
| 1 | 0.2564 | 0.1939 | 0.2327 | 0.2415 | 0.2376 | 0.2630 |
| 2 | 0.2018 | 0.2336 | 0.2076 | 0.2134 | 0.2490 | 0.2198 |
| 3 | 0.2272 | 0.2048 | 0.1832 | 0.2365 | 0.2449 | 0.2423 |
| 4 | 0.2241 | 0.2622 | 0.2263 | 0.2058 | 0.2412 | 0.2466 |
| 5 | | 0.2100 | 0.2058 | 0.2272 | 0.2286 | 0.2501 |
| 6 | | 0.2400 | 0.2421 | 0.2265 | 0.2436 | 0.2061 |
| 7 | | | 0.2395 | 0.2276 | 0.2410 | 0.2325 |
| 8 | | | 0.2696 | 0.2205 | 0.2219 | 0.2331 |
| 9 | | | | 0.2540 | 0.2192 | 0.2058 |
| 10 | | | | 0.1887 | 0.2210 | 0.2114 |
| 11 | | | | | 0.1964 | 0.2537 |
| 12 | | | | | 0.2224 | 0.2247 |
| 13 | | | | | 0.2303 | 0.2367 |
| 14 | | | | | 0.2349 | 0.2296 |
| 15 | | | | | | 0.2136 |
| 16 | | | | | | 0.2152 |
| 17 | | | | | | 0.2380 |
| 18 | | | | | | 0.2070 |




(a) Frame F4 (b) Frame F6

(c) Frame F8 (d) Frame F10

(e) Frame F14 (f) Frame F18

Fig. 11 Incremental dynamic analyses of the frames considering uncertainties in the cumulative plastic rotation capacity

concluded that reasonable estimates of structural damage can be obtained through the consideration of median cumulative rotation capacities.

Fig. 12 shows the local damage distribution in the beams of frame F10 for an $I_{DEN}$ of one. While the

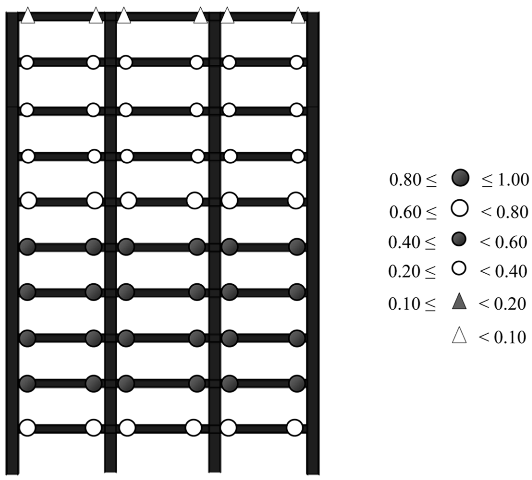



Fig. 12 Local damage distribution in Frame F10, $I_{DEN} = 1.0$

level of damage in all the beams within a story is practically the same, the value of $I_{DEN}$ of one theoretically implies structural failure at the critical stories of the frame. Note that according Fig.12, the beams located from the second to the fifth story can be considered to have failed locally. It can be concluded that $I_{DEN}$ is a useful global parameter that correlates very well to the level of damage in the critical stories of steel frames, and thus, that it can be considered as a promising tool for practical structural damage evaluation of structures subjected to large energy demands.

## 5. Conclusions

An energy-based damage model for multi-degree-of-freedom steel structures has been proposed. The model is based on the demand-supply balance of normalized plastic dissipated hysteretic energy. Particularly, the damage model is formulated as the ratio of the normalized plastic hysteretic energy demand to its corresponding capacity. While a value of zero for the damage model implies no structural damage, a unitary value implies failure.

The principal challenge for the correct use of the model is the estimation of the normalized plastic hysteretic energy capacity of complex structures. To achieve a reasonable estimation of this capacity, a damage distribution factor through height was proposed and calibrated. The factor was compared with the hysteretic energy distribution factor. It was observed that the normalized hysteretic energy capacity of a steel frame can be evaluated in a reasonable manner with both factors. The results suggest that in general, structural damage in regular steel frames tends to concentrate on a height that ranges from one third to one half of its total height (h/H around 0.4-0.5). Furthermore, no influence of the uncertainty in the cumulative plastic rotation capacity was observed during the structural evaluation of the steel frames.

The energy-based damage model introduced herein can be considered as a promising tool for the evaluation of the seismic performance of structures subjected to long duration ground motion. In these terms, the tool can be used for the formulation of design requirements of steel frames that may be subjected to severe cumulative plastic deformation demands. However, it must be emphasized that the damage model has only been calibrated for regular steel frames, designed according to the strong column-weak beam approach, and exhibiting fairly stiff beam-column connections. The use of such model under different circumstances requires specific case by case calibrations.

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## References

Akbas, B. (1997), "Energy-based earthquake resistant design of steel moment resisting frames", Ph.D thesis, Department of Civil and Architectural Engineering, Illinois Institute of Technology.

Akbas, B., Shen, J. and Hao, H. (2001), "Energy approach in performance-based design of steel moment resisting frames for basic safety objective", *Struct. Des. Tall Build.*, **10**(8), 193-217.

Akiyama, H. (1985), *Earthquake-resistant limit-state design for buildings*, University of Tokyo Press.

Arias, A. (1970), "A measure of earthquake intensity", *Seismic Design for Nuclear Power Plants*, Eds. Hansen, R.J., MIT Press, Cambridge, MA, 438-483.

Arroyo, D. and Ordaz, M. (2007), "Hysteretic energy demands for SDOF systems subjected to narrow band earthquake ground motions. Applications to the lake bed zone of Mexico City", *J. Earthq. Eng.*, **11**(2), 147-165.

Bojórquez, E. and Ruiz, S.E. (2004), "Strength reduction factors for the valley of Mexico taking into account low cycle fatigue effects", *13th World Conference on Earthquake Engineering*, Vancouver, Canada.

Bojórquez, E, Diaz, M.A., Ruiz, S.E. and Terán-Gilmore, A. (2006), "Correlation between local and global cyclic structural capacity of SMR frames", *First European Conference on Earthquake Engineering and Seismology*, Geneva Switzerland.

Bojórquez, E., Ruiz, S.E. and Terán-Gilmore, A. (2008a), "Reliability-based evaluation of steel structures using energy concepts", *Eng. Struct.*, **30**(6), 1745-1759.

Bojórquez, E., Terán-Gilmore, A. Ruiz, S.E. and Reyes-Salazar, A. (2008b), "Evaluation of structural reliability of steel frames considering cumulative damage", *The 14th World Conference on Earthquake Engineering*, Beijing, China.

Bojórquez, E. and Rivera, J.L. (2008), "Effects of degrading models for ductility and dissipated hysteretic energy in uniform annual failure rate spectra", *The 14th World Conference on Earthquake Engineering*, Beijing, China .

Bojórquez, E., Terán-Gilmore A., Bojórquez J. and Ruiz, S.E. (2009), "Explicit consideration of cumulative damage for seismic design of structures through ductility reduction factors", *Revista de Ingeniería Sísmica (Sociedad Mexicana de Ingeniería Sísmica)*, **80**, 31-62.

Bozorgnia, Y. and Bertero, V.V. (2001), "Improved shaking and damage parameters for post-earthquake applications", *Proceedings of the SMIP01 Seminar on Utilization of Strong-Motion Data*, Los Angeles, California.

Brescia, M., Landolfo, R., Mammana, O., Iannone, F., Piluso, V. and Rizzano, G. (2009), "Preliminary results of an experimental program on the cyclic response and rotation capacity of steel members", *Behaviour of Steel Structures in Seismic Areas STESSA*, Philadelphia Pennsylvania.

Calderoni, B. and Rinaldi, Z. (2000), "Inelastic dynamic and static analysis for steel MRF seismic design", *Behaviour of Steel Structures in Seismic Areas STESSA*, Balkema Rotterdam.

Calderoni, B. and Rinaldi, Z. (2002), "Seismic performance evaluation for steel MRF: non linear dynamic and static analyses", *Steel. Compos. Struct.*, **2**(2), 113-128.

Carr, A. (2002), *RUAUMOKO*, Inelastic Dynamic Analysis Program, University of Cantenbury, Department of Civil Engineering.

Cosenza, E. and Manfredi, G. (1996), "Seismic design based on low cycle fatigue criteria", *11 World Conference on Earthquake Engineering*, Acapulco, México.

Choi, H. and Kim, J. (2006), "Energy-based seismic design of buckling-restrained braced frames using hysteretic energy spectrum", *Eng. Struct.*, **28**(2), 304-311.

Engelhardt, M.D. and Husain, A.S. (1992), "Cyclic tests on large scale steel moment connections", Report No. PMFSEL 92-2, Phil M. Ferguson Structural Engineering Laboratory, University of Texas at Austin.

Fajfar, P. (1992), "Equivalent ductility factors taking into account low-cycle fatigue", *Earthq. Eng. Struct. Dynam.*, **21**(10), 837-848.

Fajfar, P. and Krawinkler, H. (1997), *Seismic Design Methodologies for the Next Generation of Codes*, A.A. Balkema.

Hancock, J. and Bommer, J.J. (2006), "A state-of-knowledge review of the influence of strong-motion duration on structural damage", *Earthq. Spectra*, **22**(3), 827-845.

Housner, G. W. (1956), "Limit design of structures to resist earthquakes", *First World Conference on Earthquake Engineering*, Berkeley, California.

Krawinkler, H. and Zohrei, M. (1983), "Cumulative damage in steel structure subjected to earthquake ground motions", *Comput. Struct.*, **16**(1-4), 531-541.

Krawinkler, H. and Nassar, A. (1992), "Seismic design based on ductility and cumulative damage demands and capacities", Eds. Krawinkler H, Fajfar P., *Nonlinear Seismic Analysis and Design of Reinforced Concrete Buildings*, Elsevier Applied Science, 95-104.

Park, Y.J. and Ang, A.H. (1985), "Mechanistic seismic damage model for reinforced concrete", *J. Struct. Eng. ASCE*, **111**(4), 740-757.

Popov, E.P. and Stephen, R.M. (1972), "Cyclic loading of full-size steel connections", American Iron and Steel Institute, Bulletin No. 21.

Rodríguez, M.E. and Ariztizabal, J.C. (1999), "Evaluation of a seismic damage parameter". *Earthq. Eng. Struct. Dynam.*, **28**(5), 463-477.

Rodríguez, M.E. and Padilla, C. (2008), "A damage index for the seismic analysis of reinforced concrete members", *J. Earthq. Eng.*, **13**(3), 364-383.

Terán-Gilmore, A. (1996), "Performance-based earthquake-resistant design of framed building using energy concepts", Ph.D Thesis, University of California Berkley.

Terán-Gilmore, A. and Jirsa, J.O. (2005), "A damage model for practical seismic design that accounts for low cycle fatigue", *Earthq. Spectra*, **21**(3), 803-832.

Terán-Gilmore, A. and Simon, R. (2006), "Use of constant cumulative ductility spectra for performance-based seismic design of ductile frames", *8th U.S. National Conference on Earthquake Engineering*.

Terán-Gilmore, A. and Jirsa, J.O. (2007), "Energy demands for seismic design against low-cycle fatigue", *Earthq. Eng. Struct. Dynam.*, **36**(3), 383-404.

Trifunac M.D. and Brady A.G. (1975), "A study of the duration of strong earthquake ground motion", *B. Seismol. Soc. Am.*, **65**(3), 581-626.

Tsai, K.C. and Popov, E.P. (1988), "Steel beam–column joints in seismic moment resisting frames", Report No. EERC 88/19, *Earthquake Engineering Research Center*, University of California at Berkeley.

Tsai, K.C., Wu, S. and Popov, E.P. (1995), "Experimental performance of seismic steel beam–column moment joints", *J. Struct. Eng-ASCE*, **121**(6), 925-931.

Uang, C.M. and Bertero, V.V. (1990), "Evaluation of seismic energy in structures", *Earthq. Eng. Struct. Dynam.*, **19**(1), 77-90.

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