Research Article

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Robust fractional control based on high gain observers design (RNFC) for a *Spirulina maxima* culture interfaced with an advanced oxidation process

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Abstract: In this article, the theory of fractional control and state estimation applied to biological science is studied, particularly in hybrid wastewater treatment. For nonlinear systems with stable and known states, an interconnected fractional robust control design with high gain state estimation is proposed to generate a control insensitive to nutritional perturbations originated by an advanced oxidation process in a microalgae culture. An online study is proposed for the mineralization of glyphosate and its feedback in a microalgae cultivation process where through the designed control the light dynamics is manipulated to robustly and automatically regulate the biomass signal provided by an analog sensor and nutrient estimation via state observers. In the literature, there are few results developed with real-time results. This work is a multidisciplinary study with online results where the performance and improvement of the proposed complex process are concluded.

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1 Introduction

The environmental problems created by agrochemicals in wastewater are severe in Latin American countries, particularly in northern cities of Mexico, where the overuse of agrochemicals has resulted in substantial health concerns. In cities such as Nuevo Casas Grandes Chihuahua, where dangerous pesticides such as glyphosate (GLP) are used illegally, many residents have claimed health concerns, including stomach and lung cancer [1]. GLP, also known as *N*-(phosphonomethyl) glycine, is a systemic organophosphorus herbicide with a broad spectrum of activity. The most widely used herbicide in the world is a broad-spectrum systemic organophosphorus herbicide [2]. In some Latin American countries, GLP is classified as class IV (low toxicity) and class III (hazardous substance). However, aminomethylphosphonic acid (AMPA) is the main metabolite and is more toxic and recalcitrant than GLP. Furthermore, the toxicity of a commercial formulation increases with the inclusion of excipients such as surfactants and others (polyoxyethylenamine,1,4-dioxane) [3,4]. Because pesticides in agroindustrial wastewater have resistant organic qualities, they are resistant to typical wastewater treatments and will eventually come into contact with the population, including the food produced where they were used, posing a significant public health risk [5]. Advanced oxidation technology (AOT) is characterized by the formation of highly oxidizing and non-selective radicals [6]. GLP and other organophosphates have been removed from 73, 91, 76, and 64% of the wastewater. The use of AOT such as UV/HSO_5 and UV/H_2O_2 is directly beneficial to the health of affected areas [7]. Toxic compounds such as GLP are transformed into a collection of extremely biodegradable substances through AOT-type processes, which could be used as nutritional sources for high-value microorganisms, such as some types of microalgae [8].

In UV/H_2O_2 AOP, photocatalytic activity breaks down the H_2O_2 molecule, releasing hydroxyl radicals (–OH). Compared to other AOTs, such as those based on the Fenton reaction, this is easy to obtain, has no mass transfer issues because of its great solubility in water, and produces no metal residues at the conclusion of the process. Furthermore, unlike heterogeneous photocatalysis, catalyst activation is required, resulting in reduced economic and operational expenses [9]. The phosphorus and nitrogen of the refractory organic compounds undergo chemical speciation changes in the AOT in addition to removal of organic materials. This allows the recovery of nitrogen and phosphorus, which, when converted into biodegradable or mineralized by-products, can be used as a source of nutrients for the development of microalgae [10–12]. *Spirulina maxima* is a blue-green microalgae that can be used in biosorption and bioaccumulation techniques to remove heavy metals from water and wastewater. *Spirulina maxima* has been grown photoautotrophically for a long time in wide fields with low cell concentrations (typically 0.4 g/L). On organic carbon substrates, this organism has recently been found to develop heterotrophically and mixotrophically (photoheterotrophically; e.g., glucose) [13,14]. The development and production of algae such as *Spirulina maxima* can be described using a variety of mathematical models: from the most basic unstructured mathematical models, such as the one illustrated in ref. [15], to the most basic ones such as Monod [16,17]. These models will be utilized to create automation theories based on the usage of nonlinear controllers and state estimators, and these approaches will be used to increase the yield and quality of microalgae cultures in the case of probable nutrient reuse using by-products from the AOT process.

To ensure superior growth management of Spirulina maxima, robust control regulations have been established. These were created by deriving robust control laws using non-linear PI controllers, fractional sliding mode, and chained observers [18–20]. This was based on the experimental working hypothesis that the dilution ratio is the only control input. However, research published in ref. [13] has revealed that light intensity has a significant impact on the Monod spectral growth parameter, suggesting that light intensity might be used as a second control input. In this study, the influence of light intensity on the growth of microalgae is automated to reject and compensate for the extra nutrients produced from the nutrient feedback derived from the AOT method. However, because most studies in the literature on automated microalgae culture are based on numerical simulation processes, this study shows experimental findings from the automation of a hybrid AOT process and microalgae cultivation. Because few commercial sensors are known to exist for the online monitoring of nutrients such as nitrates and phosphates, the controller was based on a state observer. These are approximated using observers such as those provided in ref. [21] at the base of the biomass algae sensor [22]. As a result, a multidisciplinary study is proposed in which doubly sustainable results are presented, allowing mitigation of the negative effects of pesticides such as GLP and recovery of nutrients such as nitrates and phosphates that serve as nutrient sources for the cultivation of *Spirulina maxima* through a fully automated process using a novel fractional controller proposed in this document. The light intensity, at different and specific wavelengths, can affect in different ways the growth dynamics of the microalgae. This fact has been studied in ref. [23]. In this work, we have used white light but this can be extended to different wavelengths specifically [23]. The work is divided into five sections: The problems and a basic description of the mathematical models of the stages of the AOT process and the bioreactor model are presented in the second section; the main result of this work, the design of the robust fractional controller to regulate the lamps of the photobioreactor, is presented in the third section; the methodologies and hardware of the various experiments are presented in the fourth section; and the experiment is presented in the fifth section.

2 Statement problem and plant description

Spirulina maxima biodegrade and even mineralize by-products. Mineralization of an organic compound containing phosphorus and nitrogen is an excellent source of nutrients for some types of microorganisms, such as microalgae [24,25]. Therefore, some harmful substances can be transformed into nutrients for the growth of microalgae. Thus, pesticides can serve as a nutrient source. In works such as ref. [26], it has been possible to find the ideal process times and concentrations for the mineralization of organophosphorus pesticides. These types of substances are very harmful because they are very molecularly stable and therefore highly pollutant. Therefore,

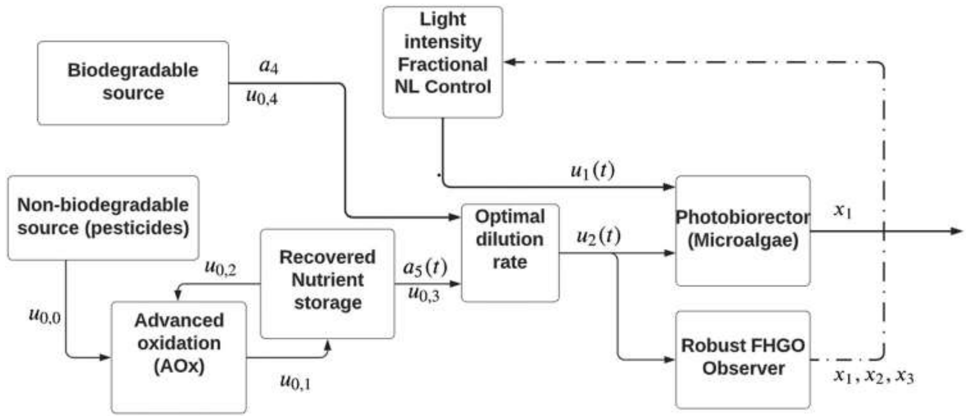

Figure 1: Schematic diagram of the process and the problem to be solved.

advanced oxidation processes are an excellent tool to transform them into assimilable nutrients for microalgae cultures in automated processes (Figure 1).

2.1 A model of an AOT stage

The recovery of nutrients derived from AOT is represented in the term $a_5(t)$; this is shown in the following mathematical model:

$$\begin{aligned}\frac{dAO_x}{dt} &= u_{0,0}P_0 - (k_1 + u_{0,1})AO_x + u_{0,2}a_5(t)\\ \frac{da_5}{dt} &= u_{0,1}AO_x - (u_{0,2} + u_{0,3})a_5(t),\end{aligned} \tag{1}$$

where P_0 is the concentration of the non-biodegradable contaminant and AO_x is the concentration of intermediate residues to finally obtain the concentration of a_5, which represents the concentration of recovered biodegradable nutrients. It should be noted that, on the basis of some research, it is possible to propose that advanced oxidation reactions can be modeled as first-order pseudo-reactions such that k_1 (as refs. [4,17]) is a constant with units (h^{-1}). AOT pumps are represented as $u_{0,i}$. Therefore, the calculation of the concentration a_5 is the general solution of a system of linear equations, and it is as follows:

$$a_5(t) = \frac{\theta_1 \sinh\left(\left(\sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}\right)t\right)\exp\left(-\left(\frac{\theta_3+\theta_4}{2}\right)t\right)}{\sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}}, \tag{2}$$

where

$$\begin{aligned}\theta_1 &= P_0u_{0,0}(k_1 + u_{0,1}),\\ \theta_2 &= (k_1 + u_{0,1})u_{0,2},\\ \theta_3 &= k_1 + u_{0,1},\\ \theta_4 &= u_{0,2} + u_{0,3}.\end{aligned} \tag{3}$$

Inferring numerous system parameters leads to the aforementioned mathematical expression $a_5(t)$ having several parameter uncertainty, as the term P_0 maintains a high uncertainty, as well as possible changes in the k_1 and AOT pumps $u_{0,i}$, and the term of recovery of nutrients $\alpha(t) = a_4 + a_5(t)$ can be proposed as a perturbation term for the next proposed photobioreactor model (1).

Note 1 *Analysis and stability conditions*

It is easy to see that the term is limited in time since for $u_{0},_{i}$, $k_1 > 0$ *such that* (2):

$$\begin{aligned}&\forall \frac{\theta_3 + \theta_4}{2} > \sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}\\ &\exists \lim_{t\to 0}(a_5(t)) = 0\\ &\exists \lim_{t\to\infty}(a_5(t)) = 0\\ &\textit{Therefore} \quad \exists\\ &|a_5(t)|\langle \sup(a_5(t)) = \psi\rangle.\end{aligned} \tag{4}$$

Such that $a_5(t)$ is a concave function with a local maximum. Whit $u_{0,0} = \gamma k$ where $\gamma > 0$ indicates the fraction or percent of $u_{0,0}$ with respect to the first-order reaction constant. In order to propose a stable stage, it should be proposed $u_{0,1} = u_{0,2} = k$ and $u_{0,0} = u_{0,3}$ with this the maximum $\psi > 0$ is guaranteed provided that the following conditions are fulfilled

$$u_{0,0} > 2\sqrt{\frac{1}{4}k^2(\gamma^2 - 2\gamma + 9)} - 3k. \tag{5}$$

Hence, AOT maximum calculation $\psi > 0$ is a very complex process since it depends on the concave function maximum bounded dimension. Summarizing all the intermediate steps, the maximum AOT is shown below

$$\psi = \zeta_\psi(\varpi_\psi^{\kappa_\psi} - \varpi_\psi\%^{\lambda_\psi}) > 0,$$

where

$$\kappa_\psi = \frac{\theta_3 + \theta_4 + 1}{4\sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}},$$

$$\lambda_\psi = -\frac{\theta_3 + \theta_4 - 1}{4\sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}},$$

$$\varpi_\psi = \frac{\left(\sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}\right) + \theta_3 + \theta_4}{\theta_3 + \theta_4 - \left(\sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}\right)}. \tag{6}$$

2.2 A Photobioreactor Model

A bioreactor is a biologically active and controlled system that uses living organisms to perform photochemical and biochemical reactions. Because this system is susceptible to changes in ambient conditions, it is critical to maintain ideal microalgae production conditions. The following continuous-phase photobioreactor model is proposed under the premise of pH, temperature, and oxygen control. A photobioreactor is a special type of bioreactor in which the interaction with light as an energy source is extremely important. Several mathematical model proposals have been made, ranging from simple classical Monod models to very complex models. These types of reactors are the most suitable for the cultivation of microalgae in a regulated and optimal way. Reach a mathematical model that has a dynamic presence on the relationship between light distribution and biomass growth. According to the article [13], there is a proportional relationship between light intensity and the specific growth constant. As a result of this, we propose a mathematical model of the photobioreactor.

$$\begin{aligned}
\dot{x}_1 &= a_1\frac{u_1(t)}{(a_2 + x_2)}x_2x_1 - u_2^\star(t)x_1\\
\dot{x}_2 &= -a_3\frac{u_1(t)}{(a_2 + x_2)}x_2x_1 - u_2^\star(t)(x_2 - a_4 - a_5(t))\\
u_2^\star(t) &= a_{1,\text{nom}}\left(1 - \left(\sqrt{\frac{a_2}{a_2 + a_4}}\right)\right) \qquad (7)\\
a_5(t) &= \frac{\theta_1 \sinh\left(\left(\sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}\right)t\right)\exp\left(-\left(\frac{\theta_3 + \theta_4}{2}\right)t\right)}{\sqrt{\frac{\theta_3^2}{4} - \frac{\theta_3\theta_4}{2} + \frac{\theta_4^2}{4} + \theta_2}},
\end{aligned}$$

where x_1 and x_2 denote the concentration of algae and biomass nutrients and $u_1(t)$ is the average luminous intensity irradiated in the reactor as the main input variable control, respectively, the a_i is, $i = 1$, 2 and 3 are Monod's type kinetic parameters, a_4 is the concentration of input nutrients, $a_5(t)$ is the concentration of input AOT, $u_2^*(t)$ is the optimal dilution rate. According to (7), it is easy to see that the control variable of the dilution rate $u_2^*(t)$ is easily disturbed by the effect of light intensity because it is designed for a specific nominal growth constant $a_{1,\text{nom}}$, which can be expressed as the specific growth constant that promotes the maximum growth. Furthermore, this signal presents another distortion promoted by advanced oxidation. As mentioned earlier, the task of this work is to provide a robust control technique to compensate for changes due to the uncertain parameter $a_5(t)$, through control. The term $a_5(t)$ may reflect effects due to impurities and uncertainties not contemplated in the material, which would trigger a dynamic uncertainty reflected in the model. The optimal dilution rate $u_2^*(t)$ can be presented as a constant and fixed value, as in ref. [27]. In this work, it is proposed that the only control variable that regulates the biomass variable is to design a single input–single output controller (SISO), where the control variable will be the Average Irradiated Luminance Intensity $u_1(t)$. Therefore, it is proposed to use the aforementioned system.

Let the nonlinear-bilinear system be as follows:

$$\begin{aligned}
\dot{x} &= f(x) + g(x)u_2(t) + \delta(\cdot)\\
y &= Cx,
\end{aligned} \tag{8}$$

where

$$x = (x_1 \quad x_2)^T$$

also

$$f(x) = \begin{pmatrix} -u_2^\star(t)x_1 \\ -u_2^\star(t)(x_2 - a_4) \end{pmatrix},$$

$$g(x) = \begin{pmatrix} a_1\frac{1}{(a_2 + x_2)}x_2x_1 \\ -a_3\frac{1}{(a_2 + x_2)}x_2x_1 \end{pmatrix},$$

$$\delta(\cdot) = \begin{pmatrix} 0 \\ -\ a_5(t)u_2^\star(t)a_3\frac{1}{(a_2 + x_2)}x_2x_1 \end{pmatrix} y \quad C = [1 \quad 0]. \tag{9}$$

In equation (8), the perturbation $\delta(\cdot)$ is the feed originating from the AOT (equation (2)) which is uncertain but is known to be bounded. This distortion can modify the optimal biomass dynamics $x_1(t)$. Thus, the signal $u_1(t)$ should compensate for this uncertainty in the online process. As is well known, there are few online sensors that allow us to measure biomass and nutrients in real time. In this work, the use of High Gain state observers to estimate variables of biomass $x_1(t)$ and nutrients $x_2(t)$ through the output signal $y = x_1(t)$. In addition, in the next section, the main result will be presented that is oriented to the design of the main biomass variable $x_1(t)$.

3 Main result of photobioreactor robust nonlinear fractional control design (RNFC)

In this section, the main result is directed toward the design of a type of robust fractional controller specially designed for photobioreactors in the presence of perturbations, in this particular case in those that originate by a hybrid connection with an AOT process. In the case of chemical and biological isotherm systems, it is known that they are stable in nature and origin. Hence, the non-linear functions that represent them are bounded. Therefore, the task of automatic control in this type of system is to be able to regulate steady states from one stable point to another [5,18]. For the case of the general nonlinear-bilinear system shown in equation (8), the following general assumptions are made:

Assumptions 1 *Nonlinear bounded RNFC*

For the case of the nonlinear functions shown in equation (8), the following conditions are proposed:

1. The function $f(x)$ is bounded such that: $\| f(x) \| \leq f_x > 0$.
2. The function $g(x)$ is bounded such that: $\|g(x) \| \leq g_x > 0$.
3. The function $\delta(\cdot)$ is bounded such that: $\| \delta(\cdot) \| \leq \psi > 0$.
4. There exists a non-linear bounden function $v(x)$ and a steady-state process time $0 < T < t$ such that:
$$g(x) = gx - v(x),$$
where $\lim_{t \to T} v(x) = 0$.

With the primary objective of developing a robust controller insensitive to disturbances, a reference system must be proposed, which is the pure system without the presence of disturbances $\delta(\cdot)$ that originated from the AOT process. Where $u_{\text{optimal}} = u_2(t)$, this is as follows:

$$\dot{z} = f(z) + g(z)u_{\text{optimal},} \tag{10}$$

where $f(z)$, $g(z)$ terms of (10) are expressed exactly the same as those shown in equation (8) for $z = x \in R_2$. The reference system (10) also meets the conditions shown in Assumption 1, where $\| f(z) \| \leq f_z > 0$, $\| g(z) \| \leq g_z$ and $\lim_{t \to T} v(z) = 0$.

On the other hand, fractional calculus is the application of classical calculus to noninteger-order derivation and integration processes. The convolution process defines fractional derivatives (non-integer) and integrals in the time domain. As a result, they are particularly suited to express memory phenomena and have been used in a variety of scientific and technical applications [28,19]. In these studies, a *RNFC* is presented to track a photobioreactor. The most common fractional operator definitions are those of Riemann–Liouville and Caputo. In this work, the Caputo definition is utilized. The fractional integral of order $0 < \beta < 1$ of the function $\varphi(t)$ on the half-positive real axis is defined, also according to

$$I^{\beta}\phi(t) = \frac{1}{\Gamma(\beta)}\int_0^t \frac{\phi(\tau)}{(t-\tau)^{1-\beta}}\mathrm{d}\tau. \tag{11}$$

As previously stated, the definitions of fractional derivative and fractional integral cannot be employed in reality; thus, quantitative approaches based on the Grunwald–Letnikov approach are often used [29]. Also, it is vital to be able to have all the states in order to develop an RNFC control, but as it is known, this is not always attainable. It is suggested to estimate the states using state observers, which have proved to have significant benefits in estimating them, particularly the high-gain observers. As a result, the following approach is presented to estimate the non-measurable states (z_2) for the system without uncertainty (10) using the biomass signal z_1. One can check that the following change of coordinates form (10) such that $\theta = \varphi(z)$:

$$\begin{aligned} \vartheta_1 &= z_1 \\ \vartheta_2 &= \frac{a_1 z_2 z_1}{(a_2 + z_2)}. \end{aligned} \tag{12}$$

In the same sense as shown in [16], the following high-gain observer can be obtained. Hence, under an appropriate persistent excitation condition, one can show that the following dynamical system is an observer for system (8) and the underlying observation error exponentially converges to zero:

$$\begin{aligned} \mathrm{d}\hat{\vartheta}/\mathrm{d}t &= \Theta(\hat{\vartheta})\hat{\vartheta} + \sigma(\cdot) - q\Delta_q^{-1}SC^TCe_{\vartheta} \\ e_{\vartheta} &= \hat{\vartheta} - \vartheta \\ \Delta_q &= \begin{pmatrix} \frac{1}{q} & 0 \\ 0 & \frac{1}{q^2} \end{pmatrix}. \\ \Theta &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ \dot{S} &= q(S + S\Theta^T(\hat{\vartheta}) + \Theta(\hat{\vartheta})S - SC^TCS) \end{aligned} \tag{13}$$

For high gain $q \gg 1$. The high gain observer (13) in original coordinates:

$$d\hat{z}/dt = f(\hat{z}) + g(\hat{z})u_2^*(t) - \frac{\partial\varphi(z)}{\partial z} q\Delta_q^{-1} SC^T Ce_{obs}. \quad (14)$$

Fulfilling the two previous conditions for systems (8) and (10), it is possible to propose the main theoretical result in the form of a theorem in addition to its analytical proof.

3.1 Theorem 1 RNF control

Let reference system (10) and a disturbed system (8) be. These fulfill the dimensioning conditions shown in the two assumptions above. It is said that a $e(t) = x - \hat{z}$ error holds ultimate bounded stability under the presence of disturbances $\delta(\cdot)$ via the use of a $u_2(t)$ control law shown in (15).

$$\begin{aligned} u_2(t) &= k_p^T e(t) + k_i^T I^\beta(e(t)) \\ k_p^T &= (k_{p,1} \quad k_{p,2}) \\ k_i^T &= (k_{i,1} \quad k_{i,2}). \end{aligned} \quad (15)$$

3.2 Proof

Let tracking error be and its derive:

$$\begin{aligned} e &= x - \hat{z} \\ \dot{e} &= \dot{x} - \dot{z} \end{aligned} \quad (16)$$

Substituting equations (8) and (10). Based on assumptions 1 and 2, it is possible to achieve the following:

$$\dot{e} = \Lambda(\cdot) - g(x)u_2(t). \quad (17)$$

The control law in equation (15) is replaced $\dot{e} = \Lambda(\cdot) - g(x)k_p^T e - g(x)k_i^T I^\beta(e(t))$. Majorinating and using the Cauchy–Bunyakovsky–Schwarz inequality:

$$\|\dot{e}\| \le h - g_x\|k_p^T\|\|e\| - g_x\|k_p^T\| \quad \|I^\beta(e(t))\|. \quad (18)$$

Whit $\|k_p^T\| \le \max(k_p)$, $\|k_i^T\| \le \max(k_i)$ and $\forall 0 < \beta < 1$ it has as follows $\|I^\beta(e(t))\| < \beta^{-1}\int(e(t))dt$.

$$\|\dot{e}\| \le h - g_x(\max k_p)\|e\| - g_x\|k_p^T\|\beta^{-1} \left\| \int (e(t)\, dt \right\|, \quad (19)$$

where $-g_x(\max k_p) = \lambda_1$ and $-g_x\|k_p^T\|\|\beta^{-1}\| = \lambda_2$. Therefore:

$$\|\dot{e}\| \le h\lambda_1\|e\| - \lambda_2 \left\| \int (e(t)\, dt \right\| \quad (20)$$

Such that:

$$-\left\| \int (e(t)\, dt \right\| < -\|e(t)\|.$$

It is had:

$$\begin{aligned} \|\dot{e}\| &\le h - (\lambda_1 + \lambda_2)\|e\| \\ \|\dot{e}\| &\le -(\lambda_1 + \lambda_2)\|e\|\left(1 - \frac{h}{\lambda_1 + \lambda_2}\right). \end{aligned} \quad (21)$$

It is easy to see that $\forall \lambda_1 + \lambda_2 \ggg h \, \exists r > 0 \rightarrow \frac{h}{\lambda_1 + \lambda_2} = r$. Therefore (8) while RNF Control (15) has the convergence ball: $\|e\| = r(1 - \exp(-(\lambda_1 + \lambda_2)t))$ with $B_e \doteq \{e \in R^n : \|e\| < r\}$.

Thus, the error is ultimately bounded.

Note 2

It is easy to see that by adjusting very large gains $k^T p$, it is possible to guarantee that the error tends to zero despite the presence of perturbations $\delta(\cdot)$ in the photobioreactor feed. Furthermore, the fractional factor β plays a role in stability, as it increases λ_2 and helps improve convergence.

4 Materials and methodologies

The initial objective is to demonstrate how the recovery of nutrients from the AOT process affects net growth in a *Spirulina maxima* culture using an open loop. A second phase, using the RNF Control controller shown in (15), demonstrates how performance is improved even in the face of perturbations. The recovered nutrients $a_5(t)$ are coupled with the classical nutrients a4 to feed the nutrients $\alpha(t)$ to the photobioreactor through control $u_2^*(t)$ after each 60-minute delay in the treatment of the AOT process. *Spirulina maxima* cultivation technique involved feeding every hour for 10 days with AOT-derived nutrients. A schematic describing the aforementioned process is presented in Figure 2. The materials, equipment, and analytical methodologies used to perform the online procedure at this stage of the project are explained in the upcoming subsections.

4.1 Advanced oxidation process

1. Chemical oxygen demand (COD) is a measurement of oxygen equivalent to the amount of organic matter in a sample that is susceptible to oxidation by a powerful chemical oxidant. COD measurements were developed by Standard methods of colorimetric micromethod

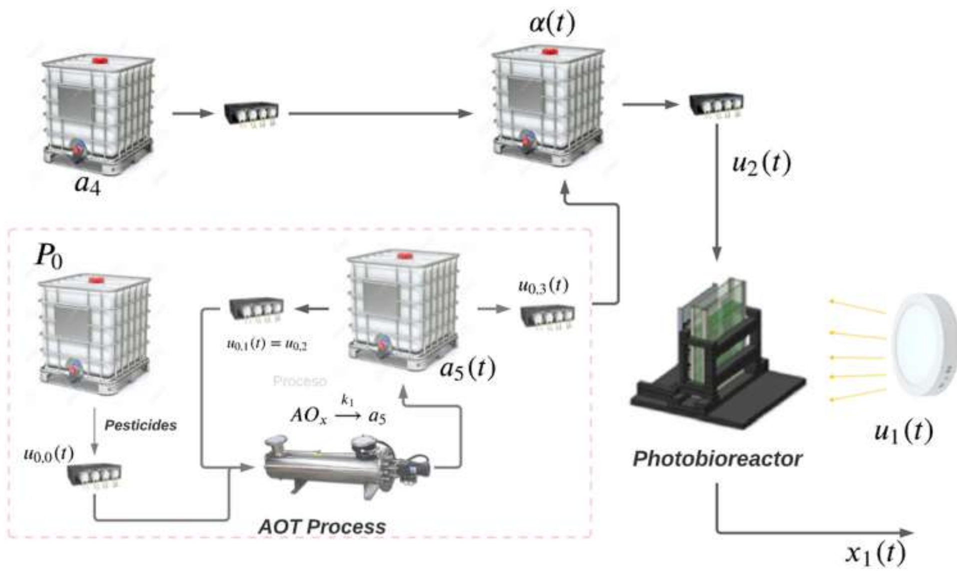

Figure 2: Schematic description of the control system.

5220 D with an ODYSSEY DR2500 spectrophotometer (Hach brand).

2. Total nitrogen was determined using the analytical technique 4500-Norg B SM, which involves digesting the sample at high temperature in an acid medium with a selenium catalyst to absorb nitrogenous organic molecules. Ammonia forms by using a catalyst to convert nitrogenous organic molecules into ammonia forms. Subsequently, distillation is used to determine the total amount of ammonium salts. The distillation of the digested material yields ammoniacal salts. It was essential to perform the following procedures using the following equation to obtain the amount of total nitrogen.
3. The total phosphorus (P) is measured in an unfiltered sample that contains all types of phosphorus. Because phosphorus can be found in conjunction with organic materials, a digestive process capable of successfully oxidizing organic matter and releasing phosphorus as orthophosphate for later use is required for the determination of total phosphorus. Phosphorus is released as orthophosphate from organic materials, which may then be determined using the ascorbic acid method. The Ascorbic Acid Method is used to determine the amount of ascorbic acid in the body. The total phosphorus content is measured in milligrams per liter [14].
4. A wastewater with Faena 320 GLP pesticide was characterized with a concentration of 320 mg/L. In this work, it is proposed to make a mixture at 120 mg/L of GLP (based on the same concentrations used in the regional agricultural industry in Chihuahua, Mexico. [1]) that will be irradiated for 60 min. A UV/H_2O_2 POA was chosen, consisting of a TFLOW stainless steel chamber and a 16 W UV light lamp with a wavelength of λwave = 220 nm and an intensity of 110 W/m2. Optical quality quartz was used to jacket the lamp, which was jacketed. The chamber had a diameter of 6 cm and a height of 37 cm. Based on previous research, a COD-H_2O_2 ratio of 0.25 was recommended for a 56.6 mg/L H_2O_2 dose to treat the GLP combination with 226 mg/L initial COD for a 56.6 mg/L H_2O_2 dosage to treat the GLP mixture with the initial COD 226 mg/L. For the dosage of H_2O_2, the recirculation of the water and the quick mixing, a peristaltic Grothen pump 12 V was used; for the water recirculation and the quick mixing, a peristaltic Grothen pump 12 V was used.

4.2 Photobioreactor stage

The typical substrate for microalgae cultivation is Zarrouk medium [30]. The cost of this medium, which is used for microalgae culture, is prohibitive for large manufacturing. For mass production, the cost of these media is affordable. Zarrouk medium is made up of expensive materials; however, it is still the best culture medium for the production of microalgae. *Spirulina maxima* biomass in the exponential phase was measured in four liters of medium containing 5% V/V *Spirulina maxima* biomass. The culture was started without nutrients recovered from the AOT process for the first part and then compared with the AOT procedure for the second part in the continuous phase. The photobioreactor was created specifically for this project. A rectangular glass cell construction 30 cm × 30 cm × 10 cm was presented with 900 Lux LED irradiation lamps suggested with airlift mixing. Based on the literature [4], the first-order pseudo-constant of this type of process is about $k = 0.20\ \mathrm{h}^{-1}$. Also, based on that, it is shown in Note 1, all dilution rates $u_{0,i} = 0.20\ \mathrm{h}^{-1}$ for the stage of the AOT process and with this for feedback of recovered nutrients for a robust online feedback process of the photobioreactor. A regulated pH of 9 and a temperature of 30 degrees Celsius are recommended for optimal growth conditions [31]. Biomass was measured using the proportional relationship between the net mass of algae and the spectrophotometric ratio at 550 nm, chlorophyll was quantified using the method [32,22], and nutrients were measured using the previously proposed direct arithmetic sum of total nitrogen and total phosphorus.

4.3 Hardware and software

Matlab 2019 is planned as a computational calculation engine that will be interfaced with Arduino Mega microcontrollers. Two PI controllers for pH and temperature are presented, based on the injection of alkaline and acid mixes for pH maintenance and a heat flow controller circuit, using Atlas Scientific pH and dissolved oxygen sensors.

It proposed a network of ten ambient luminosity sensors based on analogical sensors of type TemT 6000 to measure the luminous intensity, and an interface circuit with amperage control through Arduino Mega interface to control the luminous intensity of the LED lighting.

5 Experimental results

5.1 Open loop results

The experimental results achieved by non-feedback cultivation are provided in this section of the article. According to the literature [13], the parameter a_1 is obtained for the luminous intensity based on lux. The mathematical models of the parameters a_2, a_3 were obtained using least squares techniques based on the experimental data reported in the previous section. Parameters a_4 are obtained from total nutrients in the Zarrouk medium. The dilution rate is calculated based on those shown in the mathematical model (7). The nominal model and the parameters are shown:

$$\begin{aligned} \dot{x}_1 &= 0.000884\frac{u_1(t)}{(800 + x_2)}x_2x_1 - 0.02825x_1 \\ \dot{x}_2 &= -2.5a_1\frac{u_1(t)}{(a_2 + x_2)}x_2x_1 - u_2^{*}(t)(x_2 - 320) + \delta(t) \\ u_1(t) &= 800 \quad Lux. \end{aligned} \tag{22}$$

Based on the calculation of the optimal dilution rate shown in equation (7), we have a constant dilution rate u_2^{*}. It can be seen that the effect of recovering nutrients from the AOT process is conclusive in an increase of 278% of biomass at 240 h of the process, as demonstrated in Figures 3 and 4.

It is simple to compare the mathematical model for the nominal case and examine the influence of nutrient recovery on the production of *Spirulina maxima* using experimental data. With a fixed (open loop) $u_1(t)$ of 800 lux, the aforementioned is possible. Therefore, the presence of excess nutrients *Spirulina maxima* acts as a

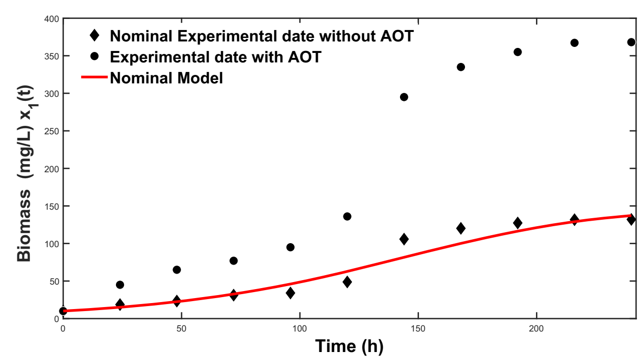

Figure 3: Comparison of biomass with/without AOT recovered substrate feedback and the nominal model.

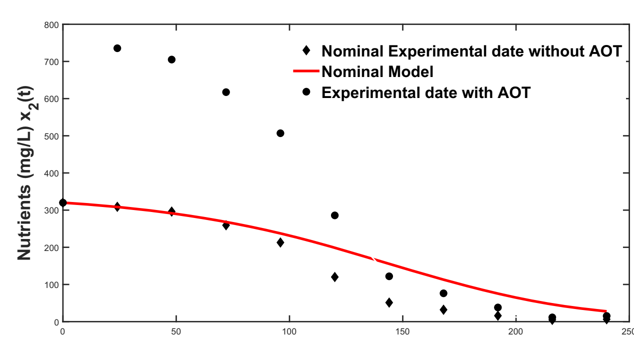

Figure 4: Comparison of nutrients with/without AOT recovered substrate feedback and the nominal model.

microalga of eutrophication, as evidenced by increased growth, demonstrating that nutrients rich in bioavailable nitrogen and phosphorus derived from oxidation are highly efficient for cell development. A maximum chlorophyll concentration of 0.35 mg/L was obtained. In prefeeding, AOT was increased by almost 130% with 0.45 mg/L using analytical methods. The results of using the RNFC controller to sustain biomass growth in a controlled growth dynamic *Spirulina maxima* using the $u_1(t)$ parameter will be shown in the next section.

5.2 Use of the RNFC control in on-line conditions

The main experimental results are presented in this section of the article, since they involve the use of a slight biomass sensor of the type shown in [22], nutrients signals are estimated via high-gain observer (12), and the robust controller RNFC (15) is used to modify dynamics $u_1(t)$ to control biomass dynamics x1(t), which have been perturbed in anticipation of the extra nutrients $a_5(t)$ (shown in (7) recovered from the advanced oxidation process of GLP quasi-mineralization). The use of a biomass sensor is suggested on the basis of measurements of three regions of the radiation spectrum: infrared, full visible, and ultraviolet. The wavelengths are 940 nm, 400–760 nm (white light), and 320 nm. On the basis of an experimental calibration, an estimate of the online biomass measurement may be obtained. It is feasible to automate the process using this signal and the calibration of the pumps and LED lamps (actuators: $u_{o.i,}$ $u_2^*(t)$ and $u_1(t)$) using an experimental online platform focused on the Matlab–Simulink language. The RNFC controller was built on this software platform to monitor the net influence on the growth and management of the biomass variable. This experimental platform is simple and inexpensive, as it uses Arduino One microcontrollers as digital and analog signal acquisition cards. A minimal scheme is shown in Figure 5.

The high-gain observer (12) proposed in the equation was designed for the nominal system; this was based on the mathematical model together with the parameters obtained and shown in the previous section (22), where the high-gain parameters are q = 10. The controller stage is proposed using an RBF control (15) using vector gains $k_p = [500\ 500]^T$ and an integral gain $k_i = [200\ 200]^{\mathrm{T}}$. Furthermore, it is proposed to use five fractional integration factors of $\beta = 0.1$, 0.3, 0.75, and 1. Therefore, this experiment was carried out four times under the same conditions, to compare the effect of fractional action on biomass control and regulation by $u_1(t)$ in the presence of nutritional pesticides based on advanced GLP oxidation. Recall that for the particular case of $\beta = 1$ of (15), there is a classical non-linear PI controller as shown in [18]. The data provided in Figures 6 and 7 indicate how the four control signals respond to changes in the constant β. Because the three signals are similar, a zoomed-in result is displayed. It can be observed that the biomass sensor performs well in measuring the biomass variable x1(t), but injects a lot of analogue noise despite using first-order filter signals.

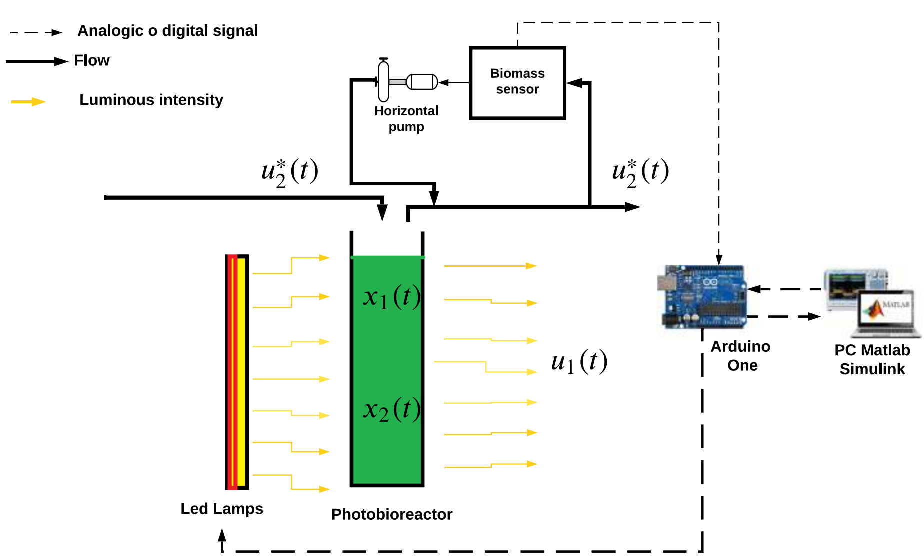

Figure 5: General scheme of lux control and regulation.

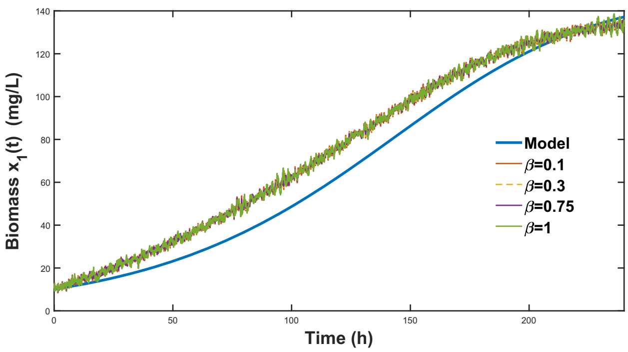

Figure 6: Comparison of the biomass dynamics in the presence of the four different controllers with a change of the beta.

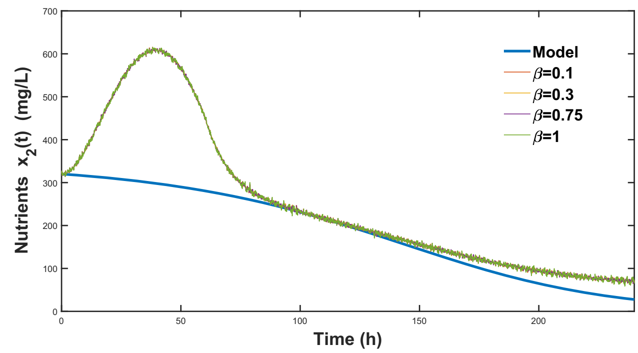

Figure 8: Comparison of the Nutrients dynamics in presence of the four different controllers with a change of the beta.

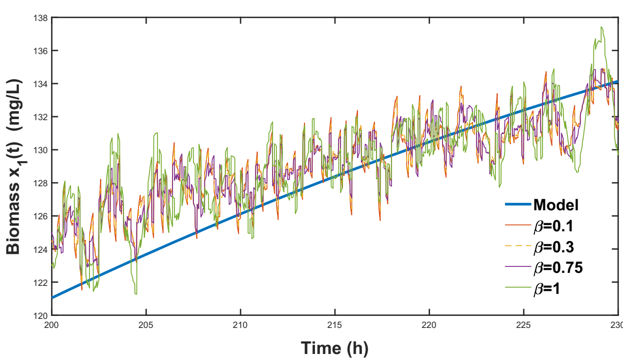

Figure 7: Comparison with Zoom of the biomass dynamics in presence of the four different controllers with a change of the beta.

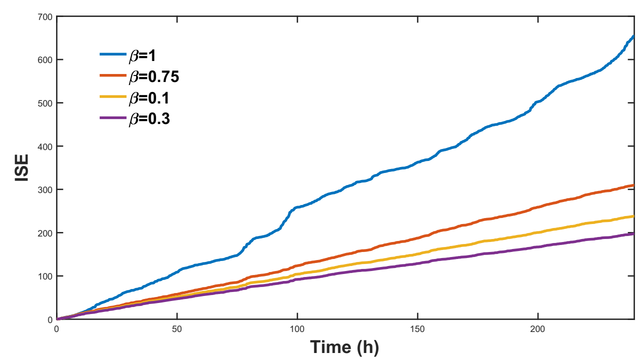

Figure 9: Comparison ISE error criteria of the four different controllers with a change in the β.

The signals of nutrients or substrates $x_2(t)$ are measured using high-gain observers given in equation (11), and the experiments mentioned in the technique section were used to validate these observations. Since there is a minimal statistical difference between them, it only shows those with $\beta = 1$ on the graph. It is possible to see a good performance in the estimate of total nutrients $x_2(t)$ in Figure 8. An investigation of the performance of the ISI and ISE indices is proposed to compare which fractional value provides the best results in the online experiment to objectively evaluate the true effect of the β meter. This form of error rate has been offered as a benchmark for comparing control rules in various processes [33–37].

$$\begin{aligned} \text{ISI} &= \int u^2 \mathrm{d}t \\ \text{ISE} &= \int e^2 \mathrm{d}t. \end{aligned} \tag{23}$$

The findings provided in Figures 9 and 10 clearly demonstrate that optimal performance is achieved for a fractional increase $\beta = 0.3$ compared to the other situations $\beta = 1$.

When comparing the dynamic errors of controllers, the reference value is typically the value of zero. As a result, it is evident that the fractional integration constant of $\beta = 0.3$ applies in both circumstances, as shown in Figures 9 and 10. This is because the dynamic error $e(t)$ is the arithmetic difference between the nominal dynamics of the model and the actual measurements obtained in the different experiments. The visible consequence of the action of the β parameter is on the dynamics of the control signal $u_1(t)$, as seen in Figure 11, since the lamp maintains a less oscillating dynamic as the fractional parameter is decreased, which helps the actuator's temporary technical health (LED lamps). This is both economically and environmentally beneficial. The dynamics of the lamps (actuators) can be affected in very lengthy process periods, as the one depicted in this study, by the oscillations in the actuators and main state $x_1(t)$ indicated in Figures 7 and 11. The controller suggested in this article significantly improves the oscillations, as shown in the image, as shown in ref. [38]. Additionally, there is a significant effect on the ISE and ISI impact, which is directly tied to the amount of energy utilized and consequently to

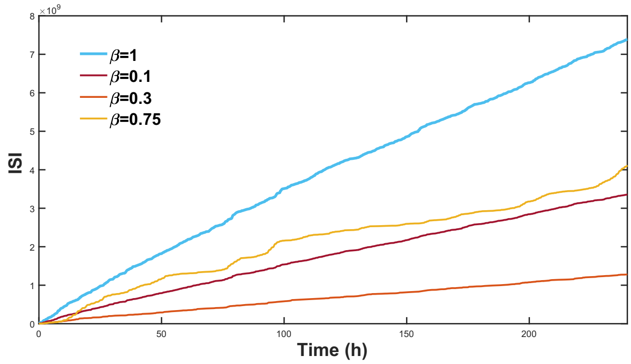

Figure 10: Comparison ISI error criteria of the four different controllers with a change in the β controllers with a change in the β.

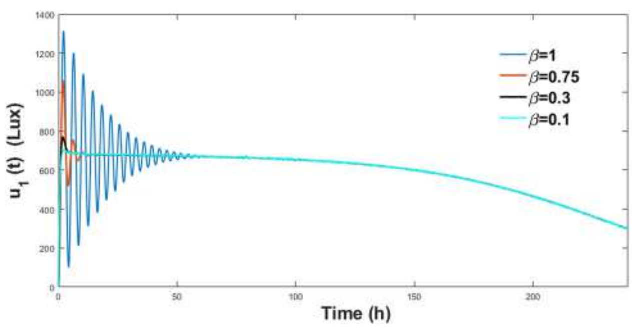

Figure 11: Comparison input u_1(t) of the four different controllers with a change in the β.

financial savings [39,40]. Therefore, in long-duration processes like the one illustrated in this article, the proposed controller is economically practical. The key feature of the controllers suggested in this research is that they significantly reduce bulb oscillations to reach the appropriate degree of control, as shown in Figure 11. As demonstrated in some research [41], such a decrease provides economic benefits because the energy occupied in a larger amplitude oscillation grows geometrically; in our particular case of this research, a saving of 56.7% was achieved compared to the PI with the fractional control $\beta = 0.3$, which had the best performance in the ISE index, as shown in Figure 9.

6 Conclusions

This study demonstrates how control theory can be used in biological sciences. Under the Lyapunov notion, a fractional PI-type control rule is proved for nonlinear systems with known steady states, which show strong tuning requirements to attain a degree of stability proven in the presentation of Theorem 1. Although it has uncertainties in nutrient delivery due to advanced oxidation and temporal noise, the control demonstrated strong resistance to such distortions, as seen in the numerical example presented in this article. It may be evaluated with online results for the growth of *Spirulina maxima* in a Matlab–Arduino context. This type of control is appropriate for systems where the mathematical model contains uncertainty, such as bioreactors, reactors, or chemical processes.

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Ethical approval: The conducted research is not related to either human or animal use.

Data availability statement: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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