Modeling of Effective Moisture Diffusivity in Corn Tortilla Baking

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Abstract: The objective of this work was to model the mass transfer in corn tortilla baking using different approaches for effective diffusivity based on the Fick's law of diffusion and to evaluate the impact of the process on quality parameters. The 1st one assumes constant effective diffusivity (method of slopes by subperiods and method of successive approximations) and the 2nd one considers variable effective diffusivity (quadratic function of time and Weibull distribution). In addition, the Weibull distribution was applied to fracturability. The effective moisture diffusivity inside the tortilla during baking is not constant and the estimations generated when considering variable diffusivity with quadratic time and Weibull distribution showed better fits (both, $R^2 = 0.999$) to the average moisture content. Quality parameter fracturability was affected by the baking process and the Weibull model adequately described ($R^2 = 0.996$) the fracturability behavior. This work will allow an adequate estimation of the concentration profiles and histories for mass transfer operations in products that can be represented as an infinite plate. The obtained analytical solutions with variable diffusivity will help to estimate the optimal conditions of the baking process to achieve the required final moisture content for baked corn tortilla shells.

Keywords: baking, corn tortilla, effective moisture diffusivity, fracturability

Practical Application: The analytical solutions of the Fick's law of diffusion for the moisture content in products that can be represented as an infinite plate, considering variable diffusivity, can be useful in studies when accurate estimations of effective diffusivity and concentration are needed.

Introduction

Corn tortilla is the main basic food in Mexico and Central America. There is currently a consumption of 800 million of tortillas per day in Mexico (Vaca-García, Martínez-Rueda, Mariezcurrena-Berasain, & Dominguez-Lopez, 2011). Corn and tortilla chips have become the most important snack food in the world (De la Parra, Serna Saldivar, & Liu, 2007). However, after deep-fat frying, these products retain up to 40% fat, and on the other hand, nowadays, health-conscious consumers demand low fat snack foods (Serna-Saldivar, 2016). An alternative to frying is baking, as no fat is added to the final product. Baked corn tortilla shells, also known as *tostadas*, are relatively new productss in the snack market and their popularity has increased due to their low fat and energy content (Palazoglu, Savran, & Gokmen, 2010). Some studies have examined combined drying technologies and others have studied processes that only imply drying or baking to produce snack chips (Kayacier & Singh, 2003; Kayacier & Singh, 2004; Xu & Kerr, 2012). Baked tortilla shell processing involves moisture transfer by evaporation from the food to the surrounding air (Fellows, 2017). Chemical, rheological, and structural changes occur inside the food, and many of these changes are function of temperature, moisture content and time (Fellows, 2017; Seth & Sarkar, 2004). Moisture content, while these changes occur, has a very important impact on the product final properties; therefore, it is relevant to evaluate the mass transfer inside the food during processing (Kayacier & Singh, 2004).

Some studies involving drying or baking of corn tortillas to generate tortilla chips have shown that there is no evidence of a constant rate period (Xu & Kerr, 2012). Generally, molecular diffusion determines the moisture transfer rate during the falling rate period (Seth & Sarkar, 2004).

As a rapid approach, from a design point of view, moisture diffusivity can be expressed as a function of the baking time; after considering that the diffusion coefficient depends on several factors as temperature, moisture content, degree of shrinkage; and all these parameters vary with baking time (Seth & Sarkar, 2004).

For describing the moisture transport rate, an effective diffusivity coefficient can be used to simplify mass transfer, which incorporates all the mechanisms involved in moisture migration during the baking of tortilla chips. These mechanisms include liquid diffusion, surface diffusion, capillary forces, Knudsen diffusion through long pores, water vapor diffusion due to vapor pressure gradients, and vapor flux due to total pressure differences. In several researches the Fick's law of diffusion has been used, considering a constant effective diffusivity (Vega-Gálvez et al., 2010; Xu & Kerr, 2012) and in others, semiempirical models were used to describe moisture transport in tortilla chips (Kayacier & Singh, 2004; Xu & Kerr, 2012). In the latter research, both, the solution of the Fick's law of diffusion considering a constant effective diffusivity and an empirical model were applied. The constant diffusivity approach

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estimated lower moisture content for the most part of the baking, and the empirical model, at the beginning of the process, predicted lower moisture contents, and later, it over predicted the moisture content of samples (Kayacier & Singh, 2004). Additionally, the empirical model does not allow the calculation of the diffusivity and its behavior.

In most mass transfer studies, during food processing, the diffusivity is not constant. Therefore, the effective diffusivity, and its behavior, when considering a constant diffusion coefficient, may not be accurately estimated. The incorporation of a variable diffusivity in an analytical model can allow better estimations of the actual concentration profiles and histories. Moreover, modeling allows choosing the most suitable operating conditions, either to dimension the baking equipment or to minimize the processing times according to the desired final specifications of the product. In this sense, optimization of the operating conditions requires effective models, particularly for heat sensitive materials such as foods (Vega-Gálvez et al., 2010). The objective of this research was to model the mass transfer in corn tortilla baking using different model approaches for effective diffusivity based on the Fick's law of diffusion and to model fracturability during the process.

Materials and Methods

Experimental methods

Preparation and baking of corn tortilla. Tortillas were prepared from traditional nixtamalized corn flour, which was mixed in the ratio 1:1 with water in an industrial mixer (Tecnomaiz, M-40, Mexico) for 120 s to form dough with 50% of moisture. The thickness of dough was adjusted in a roller machine (Rodotec, RT-100 T, Mexico) and flat disks were cooked at 270 °C, with 30 s of residence time. Corn tortillas had a nominal diameter and thickness of 14 cm and 1 mm, respectively. Selection was based on criteria of homogeneity in diameter, thickness and mass. Baking of corn tortillas to manufacture low-fat tortilla shells was carried out in a commercial electric oven (Oster, 6081-013, Mexico) at 180 °C.

Mass transfer kinetics during baking. The mass of tortilla was recorded every 5 s using an electronic balance (Sartorius, TE1502S, Germany), and an interface (Sartorius interface kit, model YDO 01 PT) connected to a computer. The average moisture content ($\overline{C}$, kg water/kg dry solid) was calculated with $\overline{C} = (m - m_{ss})/m_{ss}$ (Lara, Cortés, Briones, & Perez, 2011) where m is the mass of the sample (kg), and m_{ss} is the mass of dry solid (kg d. s.). The moisture content at the beginning of the process was determined by the method of AOAC (2012). The equilibrium moisture content was determined when no changes were observed in the average moisture content (Geankoplis, 2013). The thickness ($2L$) of the sample was measured with a digital micrometer (Inzice, CD-8 "C). In the mass transfer analysis, for every variable 40 replicates were carried out.

Fracturability. The fracture force for tortilla during the baking process was carried out following the methodology described by Salazar et al. (2014) and Janve, Yang, and Sims (2015). Samples were analyzed in a food testing machine (Instron, 3342, USA) with a 500 N load cell and the maximum load force (N) to crack the samples was recorded. The test was carried out using an 8 mm diameter stainless steel probe with a flat cylindrical end and a platform accessory with a hollow cylindrical base with 33.5 and 16 mm external and internal diameters, respectively. The test speed was 1.0 mm/s, with a travel distance of 5 mm. The Bluehill Lite software (Version 2.23) was used to analyze the data and the results were expressed as the maximum fracture force (N).

Design of experiments. A completely randomized design was carried for fracturability. The factor was baking time (0, 120, 240, 360, 480, and 630 s). Twenty replicates were performed. Fisher's test ($\alpha = 0.05$) was used for comparison of means (Montgomery, 2012).

Theoretical modeling

Mass transfer model for the baking process. A relatively simple representation of tortilla was used considering it as an infinite slab under hot air baking. The hot air flow was parallel to the top and bottom surfaces of the tortilla. The Fick's law of diffusion is:

$$\frac{\partial C}{\partial t} = D_e \frac{\partial^2 C}{\partial x^2} \tag{1}$$

where C is moisture content (kg H_2O/kg d.s.), t is the baking time (s), D_e is effective diffusivity (m^2/s), and x is the spatial dimension (m). The x coordinate is measured from the center of the slab, which has a thickness of $2L$. The initial moisture content, C_0 was considered to be uniform and the slab was suddenly immersed in hot air with constant humidity, C_∞. The initial and boundary conditions for an infinite slab with negligible convective resistance are:

$$\begin{aligned} C(x, 0) &= C_0 \\ C(-L, t) &= C_\infty \\ C(L, t) &= C_\infty \end{aligned} \tag{2}$$

The solution for the average moisture content $\overline{C}$ of the slab is given as (Crank, 1975):

$$\frac{\overline{C} - C_\infty}{C_0 - C_\infty} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \exp\left[-\frac{(2n-1)^2 \pi^2 D_e t}{4L^2}\right] \tag{3}$$

Constant effective diffusivity.

(a) Method of slopes by subperiods

When only one term of the series solution is considered, the equation is linearized, and effective diffusivity is calculated from the slope of a $\ln[(\overline{C} - C_\infty)/(C_0 - C_\infty)]$ against time curve. When the curve was not a straight line, several falling rate periods of baking were considered, each of which was characterized by a constant effective diffusivity. Because the subsequent terms in the series solution are neglected, this method is only valid for mass transfer Fourier number, $F_o = D_e t/L^2$ in excess of 0.2 with an error of 2% in the unaccomplished concentration fraction.

(b) Method of successive approximations

For each time, the mass transfer Fourier number, F_o was solved from the solution for the average moisture content with the iterative Newton's 2nd order method, considering 10 terms of the infinite series. Once the mass transfer Fourier number was known, for each time, the effective diffusivity coefficient was obtained. In addition, a constant diffusivity coefficient was assayed taking the mean of these diffusivities.

Variable effective diffusivity.

(a) Quadratic function of time

During baking of tortilla to produce low-fat tortilla shells, the diffusion coefficient increases due to structural factors, such as pore size and pore number distributions, and pore expansion by pressurization of air and water vapor. These changes in the diffusion coefficient can be expressed through the Fick's 2nd law for one-dimensional diffusion considering an effective diffusivity with a quadratic function of time,

$$\frac{\partial C}{\partial t} = (D_0 + D_1 t + D_2 t^2) \frac{\partial^2 C}{\partial x^2} \tag{4}$$

and dT is defined as:

$$dT = \left(D_0 + D_1 t + D_2 t^2\right) dt \tag{5}$$

Here, the initial effective diffusivity is indicated by D_0; the rate of change for diffusivity varies linearly ($D'_e = D_1 + 2D_2 t$) as the baking proceeds, and the initial rate of change is D_1. The increase ($D_2 > 0$) or reduction ($D_2 < 0$) in the rate of change for the diffusion coefficient ($D''_e = 2D_2$) depends on D_2, which is related to pore size and pore number distributions around a partially gelatinized starch matrix and mainly to pore expansion by pressurization of air and water vapor.

Fick's 2nd law, the initial, and boundary conditions were normalized using the unaccomplished concentration ratio, $\psi = (C - C_\infty)/(C_0 - C_\infty)$, dimensionless spatial coordinate, $\xi = x/L$, and dimensionless time, $\tau = T/L^2$, to obtain:

$$\frac{\partial \psi}{\partial \tau} = \frac{\partial^2 \psi}{\partial \xi^2} \qquad \begin{array}{c} \psi(\xi, 0) = 1 \\ \psi(-1, \tau) = 0 \\ \psi(1, \tau) = 0 \end{array} \tag{6}$$

With the separation of variables method, the solution is $\psi = (A \cos \lambda\xi + B \sin \lambda\xi) e^{-\lambda^2 \tau}$. After applying the boundary conditions, $B = 0$, and the last equation is satisfied when:

$$\lambda_n = (2n-1)\frac{\pi}{2} \text{ for } n = 1, 2, 3, \ldots \tag{7}$$

Since Fick's law is a linear partial differential equation, then by the principle of superposition, the general solution is $\psi = \sum_{n=1}^{\infty} A_n \cos \lambda_n \xi \exp(-\lambda_n^2 \tau)$. Considering the initial condition, the A_n coefficients are evaluated in a similar way to the Fourier coefficients:

$$\int_{-1}^{1} \cos \lambda_m \xi \, d\xi = \sum_{n=1}^{\infty} A_n \int_{-1}^{1} \cos \lambda_m \xi \cos \lambda_n \xi \, d\xi \tag{8}$$

Using Equation (7), and because of orthonormality, the only nonzero term in the right-hand side is obtained when $m = n$, that is, it is equal to the Kronecker delta δ_{mn}. Therefore,

$$\psi = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos (2n-1) \frac{\pi}{2} \xi \; \exp\left[-\frac{\pi^2 (2n-1)^2}{4} \tau\right] \tag{9}$$

After integration of Equation (5),

$$T = \left(D_0 + \frac{1}{2} D_1 t + \frac{1}{3} D_2 t^2\right) t \tag{10}$$

In terms of the original variables, the moisture content as a function of spatial dimension and time is obtained for a variable diffusivity with a quadratic time behavior:

$$\frac{C - C_\infty}{C_0 - C_\infty} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos \frac{(2n-1)\pi x}{2L} \times \exp\left[-\frac{(2n-1)^2 \pi^2 \left(D_0 + \frac{1}{2} D_1 t + \frac{1}{3} D_2 t^2\right) t}{4L^2}\right] \tag{11}$$

The dimensionless *average* moisture content $\bar{\psi} = \int_{-1}^{1} \psi \, d\xi / \int_{-1}^{1} d\xi$ is:

$$\bar{\psi} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \exp\left[-\frac{(2n-1)^2 \pi^2}{4} \tau\right] \tag{12}$$

In terms of the original variables, the analytical solution for the average moisture content as a function of time for a variable diffusivity with a quadratic time behavior is:

$$\frac{\overline{C} - C_\infty}{C_0 - C_\infty} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \times \exp\left[-\frac{(2n-1)^2 \pi^2 \left(D_0 + \frac{1}{2} D_1 t + \frac{1}{3} D_2 t^2\right) t}{4L^2}\right] \tag{13}$$

From Equation (13), the dimensionless time, $\tau = T/L^2$ is written as:

$$\tau = \frac{\left(D_0 + \frac{1}{2} D_1 t + \frac{1}{3} D_2 t^2\right) t}{L^2} \tag{14}$$

For each time, τ was solved from Equation (13) with the Newton's 2nd order method, using 10 terms in the infinite series. Once the dimensionless time was known, the equation was rearranged to:

$$\frac{\tau L^2}{t} = D_0 + \tfrac{1}{2} D_1 t + \tfrac{1}{3} D_2 t^2 \tag{15}$$

The D_0, D_1, and D_2 parameters can be estimated using a linear regression.

(b) Weibull distribution

The coefficient of diffusion is related to the mean squared displacement, $(\Delta x)^2$ of a Brownian particle (Einstein, 1956) through,

$$D = \lim_{\substack{\Delta x \to 0 \\ \tau \to 0}} \int_{-\infty}^{\infty} \frac{(\Delta x)^2}{2\tau} \Phi(\Delta x) \, d(\Delta x) \tag{16}$$

where, $\Phi(\Delta x)$ is the probability density function of the displacements. In the case of porous media, $\Phi(\Delta x)$ contains information on the porous structure, such as the pore size distribution, the pore number distribution, and the tortuosity; as these factors affect the mean square displacement of the particles during the diffusion process. When the structure of the porous medium changes over time, the density function of the displacements should be considered as $\Phi(\Delta x, t)$. During baking, the predominant phenomenon is the expansion of the pores by pressurization of air and water vapor, which can be considered as a temporal scaling of the density of displacements. This allows establishing $\Phi(\Delta x, t)$ as the product

Food Engineering, Materials Science, Nano of two probability densities, one independent of time that captures the porous structure of the medium, $\Phi_\infty(\Delta x)$ and other that describes the phenomenon of pores' dilatation, $f(t)$. In this way, the diffusion coefficient can be rewritten as:

$$D = \lim_{\substack{\Delta x \to 0 \\ \tau \to 0}} \int_0^t \int_{-\infty}^{\infty} \frac{(\Delta x)^2}{2\tau} \Phi_\infty(\Delta x)\, f(t)\, d(\Delta x)\, dt \quad (17)$$

Here, $\Phi_\infty(\Delta x) = \lim_{t\to\infty} \Phi(\Delta x, t)$ corresponds to the porous structure when the maximum expansion of the pores has been reached. Denoting, $F(t) = \int_0^t f(t)\,dt$ with the property $\lim_{t\to\infty} F(t) = 1$, that is usual for the distribution functions, the diffusion coefficient can be described by $D = D_\infty F(t)$, with D_∞ obtained from the density of displacements $\Phi_\infty(\Delta x)$.

Defining a normalized diffusion coefficient, $\theta_D = (D - D_0)/(D_\infty - D_0)$, where D_0 and D_∞ are the diffusion coefficients at the beginning and end of the baking process, allows the unaccomplished diffusion ratio to be written in terms of the distribution function, $F(t)$ as:

$$1 - \theta_D = \frac{D - D_\infty}{D_0 - D_\infty} = \frac{F(t) - 1}{F(t_0) - 1} \quad (18)$$

It has been widely reported that the Weibull distribution, $F(t) = 1 - e^{-(t/\beta)^\alpha}$, with scale and shape parameters, α and β, appropriately describes the particle size and bubbles distribution; likewise, its mathematical structure is similar to the distribution of pores in foods. Applying the Weibull distribution to the normalized diffusion coefficient,

$$\theta_D = 1 - \exp\left[-\frac{t^\alpha - t_0^\alpha}{\beta^\alpha}\right] \quad (19)$$

Considering that $t >> t_0$, allows a simplification to $\theta_D \approx 1 - e^{-(t/\beta)^\alpha}$. Thus, a simpler mathematical structure is obtained to describe the diffusion coefficient as a function of time:

$$D = D_0 + (D_\infty - D_0)\left(1 - e^{-(t/\beta)^\alpha}\right) \quad (20)$$

In this way, it is possible to establish that:

$$dT = \left[D_0 + (D_\infty - D_0)\left(1 - e^{-(t/\beta)^\alpha}\right)\right] dt \quad (21)$$

and after integration, the analytical solution for the average moisture content as a function of time is obtained for a variable diffusivity using a Weibull distribution:

$$\frac{\bar{C} - C_\infty}{C_0 - C_\infty} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \times \exp\left\{-\frac{(2n-1)^2\pi^2\left[D_0 + (D_\infty - D_0)\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!(k\alpha+1)}\left(\frac{t}{\beta}\right)^{k\alpha}\right]t}{4L^2}\right\} \quad (22)$$

The Weibull distribution parameters, α, β, and the extreme diffusion coefficients, D_0, D_∞, can be estimated through a nonlinear regression.

Fracturability model. The quality parameter, fracturability has been widely studied from a qualitative point of view (Kayacier & Singh, 2003; Lara et al., 2011), recognizing that during baking the fracturability increases and shows a local maximum. This has been attributed to the formation of a sponge-like network as a result of the changes in pore size and pore number distributions, due to the rapid removal of moisture, which promotes the generation of pores that contributes to the formation of a spongy structure, but then, a later increase in pore size, leads to their collapse, and therefore, to the loss of this spongy structure (Kayacier & Singh, 2003; McDonough, Gomez, Lee, Waniska, & Rooney, 1993; Moreira, Palau, Sweat, & Sun, 1995). Because variability of the diffusion coefficient is a result of the same processes of structural changes in the food, there will possibly be a very close relationship between the normalized fracturability, $\theta_F = (F - F_0)/(F_\infty - F_0)$ and the normalized diffusion coefficient, θ_D; however, due to θ_D is predominantly affected by the pore size, rather than by the pore number, it is necessary to incorporate this latter dependence. Therefore, fracturability can be modeled as the product of both effects,

$$\theta_F = \theta_D \theta_N \quad (23)$$

the effect of pore size, θ_D and pore number, θ_N. Regarding the mathematical structure of θ_N, this should have a growth trend over time, as well as, the capacity to describe a local maximum. In this sense, a linear combination of a Weibull distribution, $\theta_m = 1 - e^{-(t/b)^a}$ and its derivative (θ'_m) is proposed:

$$\theta_N = \theta_m + Ab\theta'_m \quad (24)$$

where A is a linear combination parameter, and is multiplied by the time-scale parameter, b which allows $b\theta'_m$ to be dimensionless and to have a function range of the same order as θ_m. Because the variation of pore size and pore number is governed by the same structural dynamics, it is possible that the θ_D and θ_m distributions could have the same time-scale parameter ($b = \beta$). The scale parameter was selected as the time at which the fracture force is maximum. In this way, the parameters of $\theta_F = \theta_D\theta_N$ to be estimated using nonlinear regression are the linear combination parameter A, and the shape parameter, a.

Likewise, in the analytical solution of the average moisture content, as β was fixed, the parameters to be estimated are the Weibull shape parameter, α and the extreme diffusion coefficients, D_0 and D_∞.

Results and Discussion

The initial moisture content (C_0) was 0.625 ± 0.010 kg water/kg d.s. This result was similar to that obtained by Morales-Pérez and Vélez-Ruiz (2011) (0.67 kg water/kg d.s.). The moisture content at equilibrium (C_∞) was 0.022 ± 0.004 kg water/kg d.s.; this result agrees with that reported by Xu and Kerr (2012), who stated that for crunchy snacks, such as tortilla chips, final moisture content from 2% to 3% (w. b.) is typical. The average thickness ($2L$) of the baked tortillas was $9.48 \times 10^{-4} \pm 3.29 \times 10^{-5}$ m. This result was found in the range studied by Braud, Moreira, and Castell-Perez (2001) (8.9×10^{-4} to 1.29×10^{-3} m) and Xu and Kerr (2012) (8×10^{-4} to 2.3×10^{-3} m).

The estimated effective diffusivity and the experimental and predicted baking kinetics for corn tortilla, considering constant diffusivity, are shown in Figure 1. The diffusivities obtained with the method of slopes by subperiods increased as the baking process

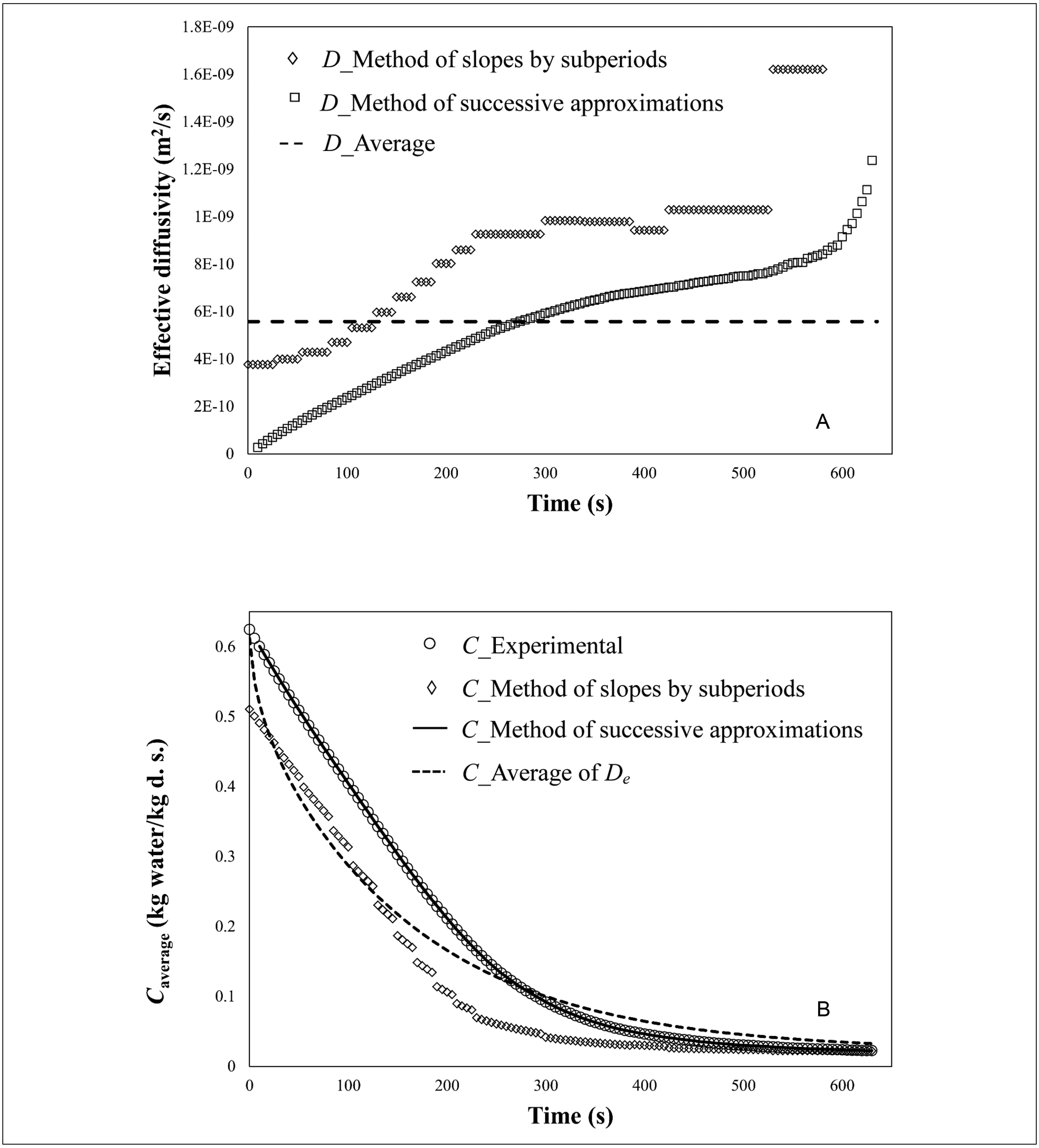

Figure 1–Estimated effective diffusivity (A), and experimental and predicted baking, kinetics (B) for corn tortillas, considering a constant diffusivity.

progressed (Figure 1A). From 0 to 25 s, the constant diffusivity was 3.76×10^{-10} m^2/s and from 620 to 630 s the constant diffusivity was 1.20×10^{-08} m^2/s. The range of effective diffusivity obtained by the method of successive approximations was from 2.66×10^{-11} to 1.24×10^{-09} m^2/s and its mean was 5.58×10^{-10} m^2/s. For each method, the corresponding diffusivities were used in the solution (Equation (3)) for the average moisture content ($\bar{C}$) considering constant diffusivity. The calculated average moisture content is shown in Figure 1(B). The method of slopes by subperiods does not predict satisfactorily during the entire baking process, because only one term of the infinite series is used and considers a constant diffusivity for each subperiod. For the successive approximations method, when substituting the corresponding diffusivities in the solution for the average moisture content (Equation (3)), the estimations were appropriately adjusted to the experimental ones, overlapping them. This is because the effective diffusivities were solved from the solution to Fick's 2nd law, which implied that when replacing them in the same solution, this

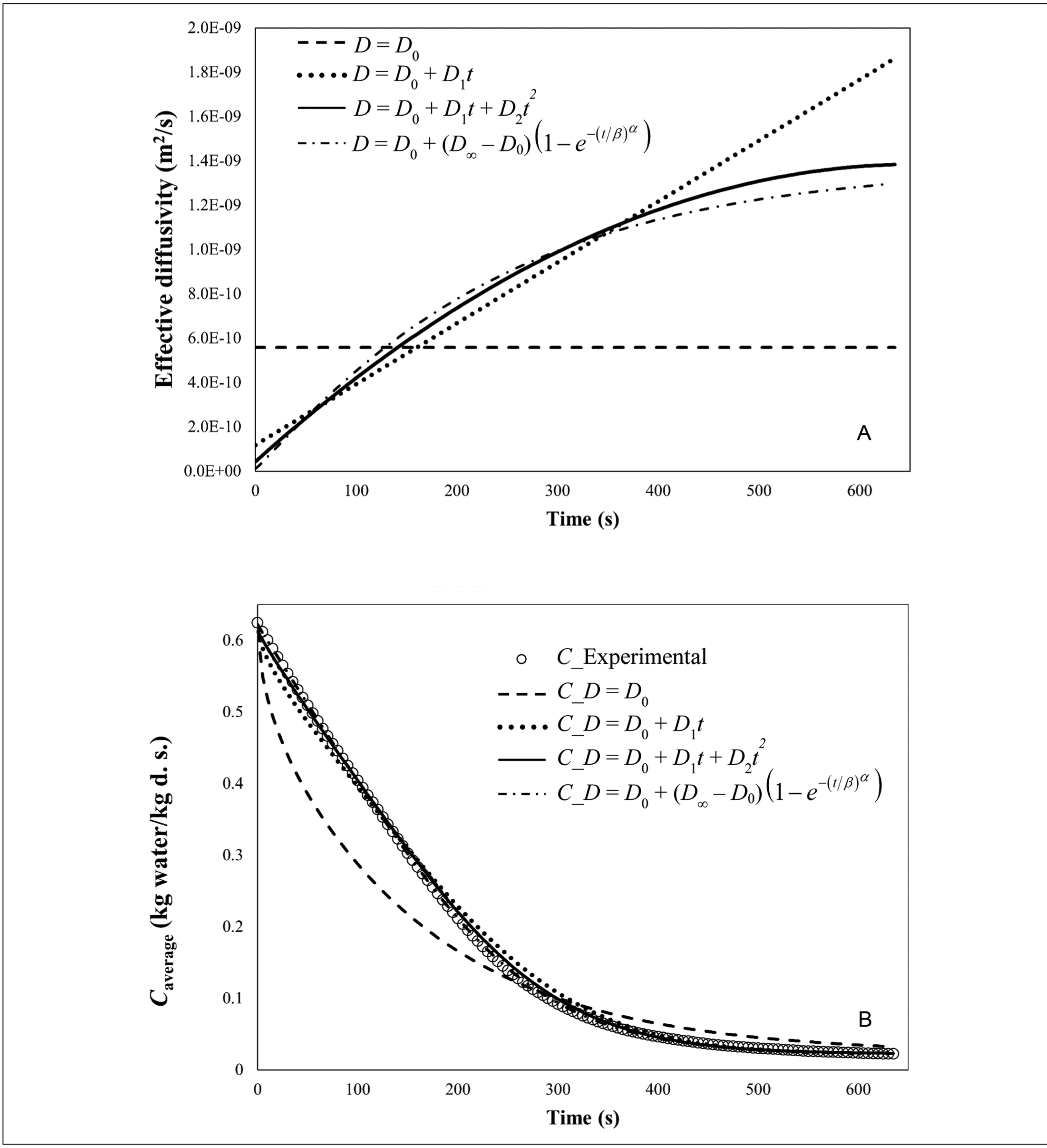

Figure 2–Estimated effective diffusivity (A) and experimental and predicted mass transfer, kinetics (B) for corn tortillas, considering variable effective diffusivity.

result was to be expected; however, as this solution assumes constant diffusivity, there is no certainty that the estimated effective diffusivities are the actual ones. On the other hand, the mean of the effective diffusivities obtained with the method of successive approximations did not predict satisfactorily the average moisture content throughout the baking process. Therefore, the variable diffusivity alternative was analyzed.

Figure 2 shows the behavior of the variable effective diffusivity (Figure 2A) and the experimental and predicted baking kinetics for corn tortilla (Figure 2B). For diffusivity with a quadratic time behavior, the initial effective diffusivity was $D_0 = 4.30 \times 10^{-11}$ m^2/s and the initial rate of change $D_1 = 4.09 \times 10^{-12}$ m^2/s^2. This rate of change for diffusivity decreases ($D_2 = -3.12 \times 10^{-15}$ m^2/s^3) linearly as the baking proceeds. In addition, a diffusivity with a linear time behavior was tested ($D_2 = 0$) and D_0 and D_1 parameters were 1.11×10^{-10} m^2/s and 2.78×10^{-12} m^2/s^2, respectively. A constant diffusivity was examined ($D_1 = D_2 = 0$) and from the regression analysis, the D_0 parameter

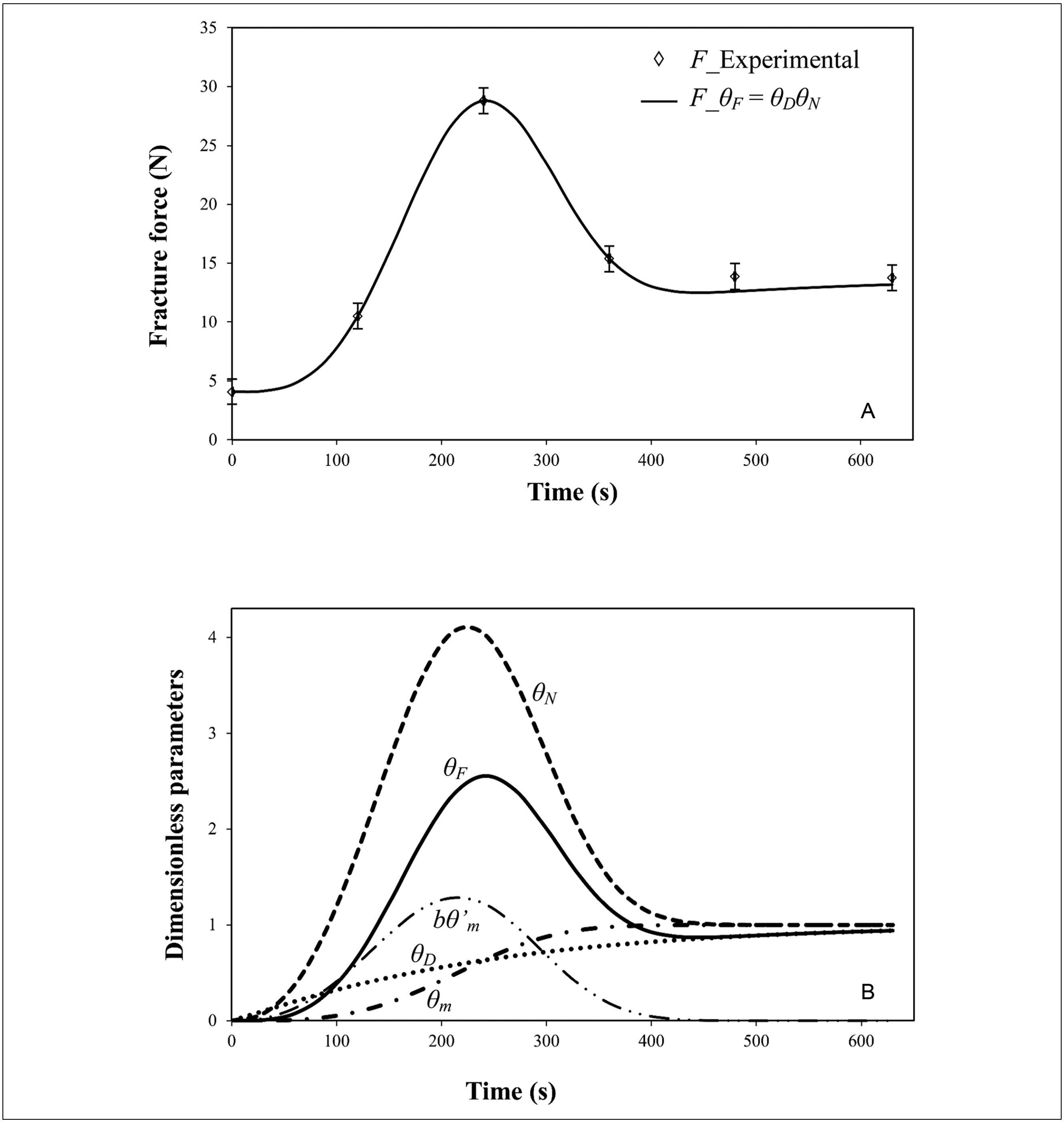

Figure 3–Fracture force, LSD = 2.18 N (A) and behavior of θ_F, θ_D, θ_N, θ_m, and $b\theta'_m$ (B), for tortilla baking to manufacture low-fat tortilla shells.

was 5.58×10^{-10} m^2/s. Finally, for diffusivity with the Weibull model, the parameters were $D_0 = 1.08 \times 10^{-11}$ m^2/s, $D_\infty = 1.38 \times 10^{-9}$ m^2/s, and $\alpha = 1.074$. The scale factor $\beta = 240$ s was the time at which the fracture force showed a maximum. Using the quadratic diffusivity model, the diffusivity increased from 4.30×10^{-11} to 1.38×10^{-9} m^2/s, with the linear model, from 1.18×10^{-10} to 1.86×10^{-9} m^2/s, when the constant diffusivity model was used, the effective diffusivity was 5.58×10^{-10} m^2/s, and using the Weibull model, diffusivity increased from 1.08×10^{-11} m^2/s to 1.30×10^{-9} m^2/s. Assuming a constant diffusivity model, Equation (13) is transformed into Equation (3), in this way, with a regression analysis from Equation (15), the same diffusivity as the mean of diffusivities estimated by the method of successive approximations (5.58×10^{-10} m^2/s) was obtained, and therefore, the same average moisture content (Figure 2B), which complies with the correspondence principle. In addition, Figure 2(B) shows that replacing diffusivities with a linear behavior in the analytical solution (Equation (13)), the estimated moisture content had an adequate fit ($R^2 = 0.996$) to the experimental data. However, the prediction generated using diffusivity with quadratic time behavior and Weibull distribution showed better fits to the experimental data (both, $R^2 = 0.999$), this is due the diffusivity inside the tortilla behaves variably during baking. Therefore, the method of successive approximations did not provide adequate diffusivities, since it assumed that the diffusivity is constant for each time.

Effective diffusivity obtained for baked tortilla shells are similar in order of magnitude to those reported by Kayacier and Sing (2004) for baked chips (6.25×10^{-10} to 10.97×10^{-10} m^2/s). In addition, Xu and Kerr (2012) used vacuum belt drying for tortilla chips and found effective diffusivities from 1.19×10^{-9} to 1.54×10^{-9} m^2/s; they marked that when a constant diffusivity is considered; the diffusion model initially predicted lower moisture contents than was observed, and later, higher moisture contents. The same behavior was observed in our research when constant diffusivity was assumed.

Fracturability

Figure 3 shows the fracture force history for tortilla baking to manufacture low-fat tortilla shells. The fracture force for tortillas was 4.08 N, similar to that reported by Flores-Farías, Salinas-Moreno, Kil-Chang, González-Hernández, and Ríos (2000) (4.67 N), and at the end of the process, for baked tortilla shells, was 13.76 N, showing a maximum fracture force (28.8 N) at 240 s. This behavior was similar to that reported by Kayacier and Singh (2003), Lujan-Acosta and Moreira (1997), Moreira et al. (1995), and Matiacevich, Mery, and Pedreschi (2012), who attributed the increase in fracturability to the formation of a spongy structure due to the increase in the number of small pores by a sudden evaporation, and its subsequent loss, to the slowing of water removal causing the collapse of cell walls and the elongation of the cracks, which makes the chips weaker. The normalized fracturability model, $\theta_F = \theta_D \theta_N$ generated an adequate fit ($R^2 = 0.996$) to the fracture force (Figure 3A) with $a = 3.327$ and $A = 2.785$. The scale factor, $\beta = b = 240$ s, for modeling both, the diffusion coefficient and fracture force, indicated that changes in pore size and pore number are governed by the same time scale. The behavior of the normalized parameters is shown in Figure 3(B). The distribution of pore number, θ_N is mainly affected by the number of micropores, which can be divided into 2 contributions: the distribution of noncollapsible micropores, θ_m and the distribution of collapsible micropores, $b\theta'_m$. This predominant behavior of a microporous structure was also observed by Kawas and Moreira (2001). Regarding the shape parameters, $\alpha < a$, the distribution of the noncollapsible micropore number (θ_m) shows a logistic behavior, indicating that its behavior could be due to a process of pressurization of moisture content, in which water (as baking proceeds) changes to steam, pressurizing the porous structure and promoting the formation of more micropores, until the walls of the collapsible microporous structure are interconnected with each other. The $b\theta'_m$ term is a measure of the change of the collapsible microporous structure, and therefore, a very close measure of the internal pressure in the processes of pressurization and depressurization of the porous structure, which is linked to the formation and destruction of unstable or collapsible pores; its relative influence on θ_N is established through the A parameter. In this sense, the θ_N distribution indicates that the number of micropores grows suddenly until 210 s and then decreases until stabilizing at 450 s.

Conclusions

The effective moisture diffusivity inside the tortilla during baking is not constant and the prediction generated when considering an effective diffusivity with a quadratic time behavior and Weibull distribution showed the best fits to the average moisture content. The method of successive approximations adequately adjusted, overlapping the data; however, as a constant effective diffusivity is considered for each time, the effective diffusivities and its behavior are not adequate.

According to the proposal, the changes in fracture force and diffusivity occur because of the same processes for structural variations; therefore, the fracture force was related to the diffusion coefficient, which is predominantely affected by the pore size, and the pore number; which is mainly affected by the noncollapsible and collapsible micropore numbers. The changes in pore size and pore number are governed by the same time scale.

Considering a variable diffusivity with a quadratic time behavior and a Weibull distribution allowed to estimate adequately the average moisture content during the tortilla baking. This approach will allow an adequate estimation of the average moisture content or the concentration of a substance as a function of time in other products that can be represented as an infinite plate, considering the specific parameters (L, D_e, C_0, and C_∞) of the product under study. The obtained analythical solutions, when considering variable diffusivity models, will help to estimate the optimal conditions of the baking process (that is, time and temperature) to achieve a required final moisture content for baked tortilla shells; thereby granting acceptable characteristics for its subsequent consumption. In addition, accurate estimations of moisture content and effective diffusivity, at any time, could help to predict the dispersion in moisture content throughout the baking, to evaluate the influence of variability in the involved parameters on moisture content variability, and suggest strategies to improve the process.

Author Contributions

R. Iribe-Salazar and J. Caro-Corrales designed the study, interpreted the results, performed most of the experimental work, and drafted the manuscript. R. Gutiérrez-Dorado, E. Ríos-Iribe, and O. Hernández-Calderón participated in experiment design, manuscript guidance, and critically reviewed the manuscript. M. Carrazco-Escalante and Y. Vázquez-López participated in experimental work, contributed in the analysis and discussion of overall results, and reviewed the manuscript.

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